500+ Solved Physics Homework and Exam Problems

For Class 11 & Class 12

AP Physics Exams

and Colleges

BY

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Sections and Chapters

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Contents

1 Vectors

1.1 Unit Vectors

1. **Find the unit vector in the direction $\vec{w} = (5, 2)$.**

   Solution: A unit vector in physics is defined as a dimensionless vector whose magnitude is exactly 1.

   A unit vector that points in the direction of $\vec{A}$ is determined by formula

   $$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

   Where $|A|$ is the magnitude of the vector $\vec{A}$ with the following formula

   $$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

   To find the unit vector points in the direction of the given vector, $\vec{W}$, first find its magnitude as

   $$|\vec{w}| = \sqrt{5^2 + 2^2} = \sqrt{29}$$

   Next divide the given vector by its magnitude as below

   $$\hat{w} = \frac{(5, 2)}{\sqrt{29}} = \left(\frac{5}{\sqrt{29}}, \frac{2}{\sqrt{29}}\right)$$

   Therefore, the unit vector in the direction of $\vec{w} = (5, 2)$ is $\left(\frac{5}{\sqrt{29}}, \frac{2}{\sqrt{29}}\right)$.

2. **What is the unit vector point in the direction of $\vec{v} = (-4, -8)$.**

   Solution: The unit vector along an arbitrary vector is computed as the vector divided by its magnitude.

   The magnitude of the given vector is found by the formula below

   $$|\vec{v}| = \sqrt{(-4)^2 + (-8)^2} = \sqrt{80}$$

   Therefore, applying unit vector definition, we have

   $$\hat{v} = \frac{(-4, -8)}{\sqrt{80}} = \left(\frac{-1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}\right)$$

   where in the last step, we used the simplification $\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$.

3. **Find the unit vector in the direction of sum of two vectors $\vec{v} = (2, -4)$ and $\vec{w} = (-3, 2)$.**

   Solution: the sum of two given vectors, we call it $\vec{c}$, is calculated as below

   $$\begin{aligned}\vec{c} &= \vec{v} + \vec{w} \\ &= (2, -4) + (-3, 2) \\ &= (2 + (-3), -4 + 2) \\ &= (-1, -2)\end{aligned}$$

The magnitude of this vector is also found as

$$|\vec{c}| = \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$$

Now, according to unit vector definition, divide the obtained vector by its magnitude to find the unit vector points in the direction of sum of the two vectors given as below

$$\hat{c} = \frac{(-1, -2)}{\sqrt{5}} = \left(\frac{-1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}\right)$$

4. **Find a vector in the direction of unit vector $\hat{v} = \left(1/3, \sqrt{8}/3\right)$ and a magnitude of 11.**

   Solution: By definition, a unit vector has a length (magnitude) of 1. To check this condition for the given vector, we use the Pythagorean theorem to find its magnitude as

$$|\hat{v}| = \sqrt{(1/3)^2 + (\sqrt{8}/3)^2} = 1$$

   As expected for a unit vector. With the help of unit vector definition, we can construct a vector with an arbitrary magnitude. If you scale up or down a unit vector by a constant coefficient, then you construct a vector with the magnitude of that coefficient.

   Consequently, the vector $\vec{w} = k\hat{v}$, where k is a constant coefficient, has a magnitude of k.

   To find a vector with a magnitude of 11 and in the direction of the unit vector given above, $\hat{v}$, we simply scale up that unit vector by 11. Hence, the desired vector is

$$\vec{u} = 11\hat{v} = \left(\frac{11}{3}, \frac{11\sqrt{8}}{3}\right)$$

   where in the last step, we used the definition of multiplication of a vector by a constant coefficient (scalar) as

$$k(a, b) = (ka, kb)$$

5. **Find the unit vector in the direction of the resultant vectors $\vec{A} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\vec{B} = -\hat{\imath} + \hat{\jmath} + 2\hat{k}$.**

   Solution: The vectors are in space and their components were given. The sum of two vectors is called the resultant.

   By applying vector addition formula, the components of the resultant vector, $\vec{C} = \vec{A} + \vec{B}$ are found as below

$$\begin{aligned}\vec{C} &= \vec{A} + \vec{B} \\ &= (2, -3, 4) + (-1, 1, 2) \\ &= (2 + (-1), -3 + 1, 4 + 2) \\ &= (1, -2, 6)\end{aligned}$$

   Thus, the resultant vector is $\vec{C} = (1, -2, 6)$.

Using unit vector definition, the components of vector divided by its magnitude, the unit vector in the direction of the resultant vector is found as follows

$$\hat{c} = \frac{\vec{C}}{|\vec{C}|}$$

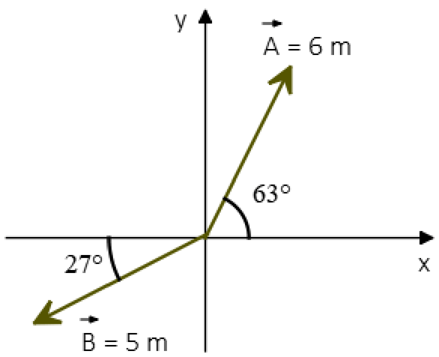
$$= \frac{(1,-2,6)}{\sqrt{1^2+(-2)^2+6^2}}$$

$$= \left(\frac{1}{\sqrt{41}}, \frac{-2}{\sqrt{41}}, \frac{6}{\sqrt{41}}\right)$$

6. **In the following figure,**

   (a) write each vector in terms of unit vectors $\hat{i}$ and $\hat{j}$,

   (b) Use unit vector definition to express the vector $\vec{C} = 3\vec{A} - 2\vec{B}$.



Solution: The notation $\hat{i}$ and $\hat{j}$ are the unit vectors (magnitude of 1) in the direction of x and y axes. Here, the magnitude and direction (angle) of the vectors are given.

(a) First, resolve the vectors into their components.

The components of a vector $\vec{A}$ with magnitude of $|\vec{A}|$ are determined by the following formula

$$\vec{A}_x = |\vec{A}| \cos\theta$$
$$\vec{A}_y = |\vec{A}| \sin\theta$$

We cal also vector sum the above components to write the vector in the usual form as below

$$\vec{A} = \vec{A}_x \hat{i} + \vec{A}_y \hat{j}$$

Therefore, the decomposition of the given vectors are

$$\vec{A}_x = 6\cos 63^\circ = 2.72\,\mathrm{m}$$
$$\vec{A}_y = 6\sin 63^\circ = 5.34$$

$$\vec{B}_x = 5\cos 27^\circ = 4.45$$
$$\vec{B}_y = 5\sin 27^\circ = 2.7$$

The relationship between components of a ordinary vector and unit vectors in the x and y directions, are as below

$$\vec{A}_x = |\vec{A}|\hat{\imath}$$
$$\vec{A}_y = |\vec{A}|\hat{\jmath}$$

Combining all things together gives the following results for the given vectors in terms of units vectors

$$\vec{A} = 2.72\hat{\imath} + 5.34\hat{\jmath}$$
$$\vec{B} = 4.45\hat{\imath} + 2.7\hat{\jmath}$$

(b) We are to multiply the vector $\vec{A}$ by 2 and subtract 2 times of vector $\vec{B}$ from the result.

According to the rules of multiplication of a vector by a scalar (a constant coefficient), to multiply $\vec{A}$ by 2, we must multiply each of its components by 2 as below

$$2\vec{A} = 2(2.72, 5.34) = (5.44, 10.68)$$

Similarly,

$$2\vec{B} = 2(4.45, 2.7) = (8.9, 5.4)$$

To subtract two vectors from each other, simply subtract its components as below

$$\begin{aligned}\vec{C} &= 3\vec{A} - 2\vec{B}\\ &= (5.44, 10.68) - (8.9, 5.4)\\ &= (-3.46, 5.28)\end{aligned}$$

Using the Pythagorean theorem the magnitude of this vector is found as

$$|\vec{C}| = \sqrt{(-3.46)^2 + (5.28)^2} = 6.31$$

To find the unit vector in the direction of the vector $\vec{C}$, we use the definition of unit vector

$$\hat{c} = \frac{\vec{C}}{|\vec{C}|}$$

$$= \frac{(-3.46, 5.28)}{6.31}$$

$$= (0.55, 0.84)$$

We leave this to you to check that the magnitude of this vector is 1.

1.2 Vector Word Problems

7. **A cruise ship travels** 200 km **due east from point** A **to** B **and then** 300 km **due south from** B **to the final destination, point** C**.**

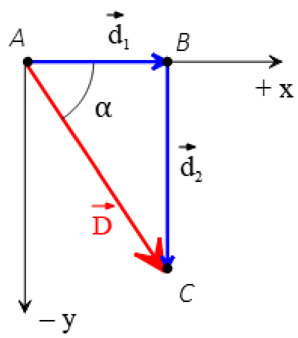
   (a) Write the displacement vector in component form.

   (b) How far is the direct distance between A **to** C**?**

   (c) What angle does the displacement vector make with the positive x**-axis?**

Solution: The goal is to find the displacement between start to finish. There are two methods to achieve this.

One is using the graphical method and applying the Pythagorean theorem to find the hypotenuse of the right triangle (which is the direct distance) as the figure below.



The second is using vectors in components form as follows.

(a) The expression of 200 km due east in vector language is written as $\vec{d}_1 = 200\,\hat{i}$ and similarly, 300 km due south is $\vec{d}_2 = -300\,\hat{j}$. Adding the above vectors yields the displacement vector

$$\begin{aligned}\vec{D} &= \vec{d}_1 + \vec{d}_2 \\ &= 200\,\hat{i} - 300\,\hat{j}\end{aligned}$$

(b) The direct distance across the whole path is found by calculating the magnitude of the displacement vector. Therefore,

$$\begin{aligned}|\vec{D}| &= \sqrt{D_x^2 + D_y^2} \\ &= \sqrt{(200)^2 + (-300)^2} \\ &= 500 \quad \text{km}\end{aligned}$$

(c) The angle that a vector makes with the $+x$ axis in the counterclockwise direction is found by the formula below

$$\alpha = \tan^{-1}\left(\frac{D_y}{D_x}\right)$$

$$= \tan^{-1}\left(\frac{-300}{200}\right)$$

$$= -56.3^\circ$$

Note that this formula does not give always the correct angle!

If the vector lies in the first and fourth quadrant, then this gets the correct angle but for vectors in the second and third quadrant, we must add 180° to the angle obtained by the formula above. In this way, you will always get the right angle.

Here, the resultant vector lies in the fourth quadrant so the negative in the angle obtained above indicates that it is below the positive x-axis.

8. **A plane flies from point A to B, a distance of $300\,\mathrm{km}$ in the direction of 32° east of north. After military operations, the plane flies to point C which is $340\,\mathrm{km}$ away, and 63° west of north.**

   (a) Write each displacement in vector components form.

   (b) Find the displacement and direction from start to finish.

   (c) What is the total distance traveled by plane?

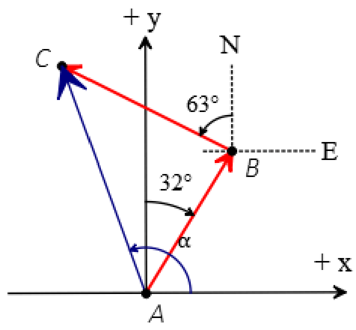
   Solution: The magnitude and direction of each flight path were given. Each direct path is assumed as a vector. In this problem, pay attention to the given angles.

   The angle in the formula of decomposition of a vector into its components is measured from the $+x$-axis in a counterclockwise direction.

$$A_x = |\vec{A}|\cos\theta$$
$$A_y = |\vec{A}|\sin\theta$$

   Here, the east of north means that you first stand facing to the north, then move as much as 32° to the east. But this angle is measured from the +y direction in the CW direction. To convert it from the +x axis, we have $90^\circ - 32^\circ = 58^\circ$.

   Similarly, 63° west of north corresponds to $90^\circ + 63^\circ = 153^\circ$ measured from the $+x$-axis in a CCW direction (figure below).



(a) With this information, the components of the vectors are written as below

$$\begin{aligned}\vec{d}_{AB} &= |\vec{d}_{AB}|\cos 58^\circ\,\hat{i} + |\vec{d}_{AB}|\sin 58^\circ\,\hat{j}\\ &= (300)(0.53)\,\hat{i} + (300)(0.85)\,\hat{j}\\ &= 159\,\hat{i} + 255\,\hat{j}\end{aligned}$$

And similarly, for the second path we have

$$\begin{aligned}\vec{d}_{BC} &= |\vec{d}_{BC}|\cos 153^\circ\,\hat{i} + |\vec{d}_{BC}|\sin 153^\circ\,\hat{j}\\ &= (340)(-0.9)\,\hat{i} + (340)(0.45)\,\hat{j}\\ &= -306\,\hat{i} + 153\,\hat{j}\end{aligned}$$

Therefore, each displacement in vector components are summarized as $\vec{d_1} = (159, 255)$ and $\vec{d_2} = (-306, 153)$.

(b) Total displacement is the sum of the two displacements above which is a vector addition problem. To accomplish this, we must sum the corresponding components with each other.

$$\begin{aligned}\vec{D} &= \vec{d_1} + \vec{d_2}\\ &= (159, 255) + (-306, 153)\\ &= (159 - 306, 255 + 153)\\ &= (-147, 408)\end{aligned}$$

Therefore, the total displacement from start to finish is written like a vector as below

$$\vec{D} = -147\,\hat{i} + 408\,\hat{j}$$

The angle that this vector makes with the positive x-axis is computed as below

$$\begin{aligned}\alpha &= \tan^{-1}\left(\frac{D_y}{D_x}\right)\\ &= \tan^{-1}\left(\frac{408}{-147}\right)\\ &= -70^\circ\end{aligned}$$

In this vector problem, the displacement vector lies in the second quadrant so the right angle with the positive x-axis will be obtained as

$$\alpha = 180^\circ - 70^\circ = 110^\circ$$

1.3 vectors

9. **Find the x and y components of the following vectors in physics**

   (a) A $10-\mathrm{m}$ displacement vector that makes an angle of 30° with the $+x$ direction.

   (b) A $20-\mathrm{m/s}$ velocity vector that makes an angle of 37° counterclockwise from the $-x$ direction.

   (c) A $80-\mathrm{N}$ force vector that makes an angle of 135° counterclockwise from the $-y$ direction.

   Solution: The length and the direction (angle) of the vectors are given. Recall that with having this information, we can relate them to the components of a vector by the following formulas

$$R_x = |\vec{R}| \cos\theta$$
$$R_y = |\vec{R}| \sin\theta$$

Where θ is the angle the vector $\vec{R}$ makes with the positive x axis and is measured in a counterclockwise direction.

(a) Here, the components are

$$\begin{aligned} R_x &= |\vec{R}| \cos\theta \\ &= (10)\cos 30^\circ \\ &= 5\sqrt{3} \quad \mathrm{m} \end{aligned}$$

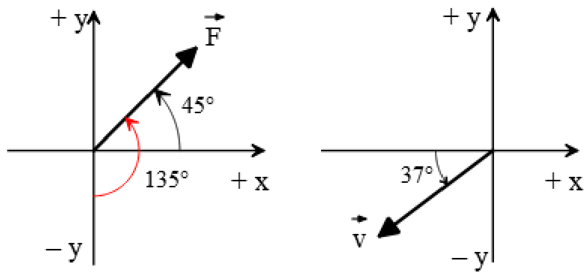
$$\begin{aligned} R_y &= |\vec{R}| \sin\theta \\ &= (10)\sin 30^\circ \\ &= 5 \quad \mathrm{m} \end{aligned}$$

(b) In this section, the angle with the negative x-axis given as shown in the figure. To measure it from the $+x$ axis in the counterclockwise direction, we must add it to 180°. Therefore, the final angle is $180^\circ + 37^\circ = 217^\circ$.

$$\begin{aligned} v_x &= |\vec{v}| \cos\theta \\ &= (20)\cos 217^\circ \\ &= -16 \quad \mathrm{m/s} \end{aligned}$$

$$\begin{aligned} v_y &= |\vec{v}| \sin\theta \\ &= (20)\sin 217^\circ \\ &= -12 \quad \mathrm{m/s} \end{aligned}$$

(c) Here, the direction of the force vector is measured from the $-y$ direction as the figure below. As you can see, the vector makes an angle of 45° with the $+x$ axis in the counterclockwise direction.



Thus, the components of the force vector are

$$\begin{aligned} F_x &= |\vec{F}|\cos\theta \\ &= (80)\cos 45^\circ \\ &= 40\sqrt{2} \quad \text{N} \end{aligned}$$

$$\begin{aligned} F_y &= |\vec{F}|\sin\theta \\ &= (80)\sin 45^\circ \\ &= 40\sqrt{2} \quad \text{N} \end{aligned}$$

10. **A vector has an x-component of -10 units and y-components of 13 units. Find the magnitude and direction of the vector.**

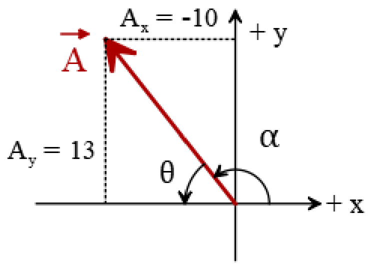
Solution: The components of a vector, say $\vec{A}$, are given which is related to the magnitude and direction of the vector by the following formula

$$\begin{aligned} |\vec{A}| &= \sqrt{A_x^2 + A_y^2} \\ &= \sqrt{(-10)^2 + (13)^2} \\ &= 16.4 \end{aligned}$$

$$\begin{aligned} \theta &= \arctan\frac{A_y}{A_x} \\ &= \arctan\frac{13}{-10} \\ &= -52^\circ \end{aligned}$$

There is a subtle point about the formula of the vector direction. If the vector lies in the first and fourth quadrants then the angle obtained from the above formula is the right answer.

Otherwise, we must add 180° to the angle obtained from the formula to get the correct angle.



In this vector problem, the components show that the original vector lies in the second quadrant, so the correct angle with the $+x$-axis in a counterclockwise direction (the standard angle measured for a vector) is obtained as below

$$\alpha = 180° - 52° = 128°$$

Consequently, the vector has a length of 16.4 units and makes an angle of 128° with the $+x$-axis.

2 Average velocity and speed

11. **A bird is flying** 100 m **due east at** 10 m/s **and then it turns around and flying west in** 15 s **at** 20 m/s**. Find the average velocity and average speed during the overall time interval.**

    Solution: First we must find the overall time. The first and second parts are done in $\Delta t_1 = \frac{\Delta x}{v} = \frac{100}{10} = 10\,\text{s}$ and $\Delta t_2 = 15\,\text{s}$. Thus the overall time of flying of the bird is $\Delta t_{tot} = 10 + 15 = 25\,\text{s}$.

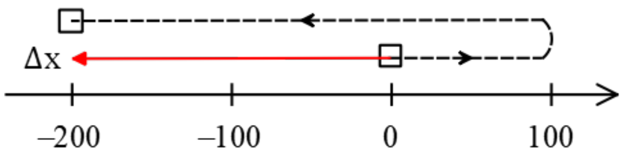
    The distance traveled of the bird due west is $d_w = v\Delta t_2 = 20 \times 15 = 300\,\text{m}$. Therefore, by definition of the average speed, we have

    $$\text{Average speed} = \frac{(100 + 300)\,\text{m}}{25\,\text{s}} = 16\ \frac{\text{m}}{\text{s}}$$

    As we can see from the figure, the displacement vector is $\Delta x = x_f - x_i = -200 - 0 = -200\,\text{m}$. So the average velocity is found as

    $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-200\,\text{m}}{25\,\text{s}} = -8\ \text{m/s}$$

    The negative indicates that the average velocity is towards the $-x$ direction.



12. **An object travels** 1200 m **in** 30 s**, what is its average velocity?**

Solution: applying the average velocity definition, we get

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

$$= \frac{1200\,\mathrm{m}}{30\,\mathrm{s}}$$

$$= 40\,\mathrm{m/s}$$

13. **An object travels $1000\,\mathrm{m}$ with an average velocity of $50\,\mathrm{m/s}$, what is the total time of travel?**

Solution: in the definition of average velocity, solving for t we get

$$\Delta t = \frac{\Delta x}{\bar{v}}$$

$$= \frac{1000\,\mathrm{m}}{50\,\mathrm{s}}$$

$$= 20\,\mathrm{s}$$

3 Average Acceleration

14. **A car is traveling in a straight line along a highway at a constant speed of 80 miles per hour for 10 seconds. Find its acceleration?**

Solution: Average acceleration is a change in velocity divided by the time taken. Since the car's velocity (magnitude and direction) is constant over the entire course, so by definition of average acceleration, it is zero i.e. $\bar{a} = 0$.

15. **A plane has a take-off speed of $300\,\frac{\mathrm{km}}{\mathrm{h}}$. What is the average acceleration (in $\frac{\mathrm{m}}{\mathrm{s}^2}$) of the plane if the plane started from rest and took 45 seconds to take off?**

Solution: Plane is initially at rest so $\vec{v_1} = 0$ and its take off speed is $\vec{v_2} = 300\,\frac{\mathrm{km}}{\mathrm{h}}$. First off, convert $\frac{\mathrm{km}}{\mathrm{h}}$ to SI units of velocity $\frac{\mathrm{m}}{\mathrm{s}}$ as below

$$300\,\frac{\mathrm{km}}{\mathrm{h}} = 300\,\frac{1000\,\mathrm{m}}{3600\,\mathrm{s}}$$

$$= 300\,\frac{1000}{3600}\,\frac{\mathrm{m}}{\mathrm{s}}$$

$$= 83.4\,\frac{\mathrm{m}}{\mathrm{s}}$$

Now ratio of change in velocity, $\Delta\vec{v} = 83.4\,\frac{\text{m}}{\text{s}}$ over time elapsed $\Delta t = 45\,\text{s}$ is definition of average acceleration.

$$\bar{a} = \frac{\Delta\vec{v}}{\Delta t}$$

$$= \frac{83.4\,\text{m/s}}{40\,\text{s}} = 2.085\,\frac{\text{m}}{\text{s}^2}$$

16. **What average acceleration is needed to accelerate a car from $36\,\frac{\text{km}}{\text{h}}$ to $72\,\frac{\text{km}}{\text{h}}$ in 25 seconds?**

Solution: Initial and final velocities are $36\,\frac{\text{km}}{\text{h}}$ and $72\,\frac{\text{km}}{\text{h}}$, respectively. As before, convert them in SI units as below

$$\frac{\text{km}}{\text{h}} = \frac{1000\,\text{m}}{3600\,\text{s}}$$

$$= \frac{1000}{3600}\,\frac{\text{m}}{\text{s}}$$

$$= \frac{10}{36}\,\frac{\text{m}}{\text{s}}$$

In other words, multiply them by $\frac{10}{36}$. Then, $v_1 = 36 \cdot \frac{10}{36} = 10\,\frac{\text{m}}{\text{s}}$ and $v_2 = 72 \cdot \frac{10}{36} = 20\,\frac{\text{m}}{\text{s}}$. Now dividing change of velocity, $\Delta v = 20 - 10 = 10\,\frac{\text{m}}{\text{s}}$ over the time elapsed $\Delta t = 25\,\text{s}$ we get the desired average acceleration

$$\bar{a} = \frac{\Delta\vec{v}}{\Delta t}$$
$$= \frac{20\,\text{m/s}}{25\,\text{s}} = 0.8\,\frac{\text{m}}{\text{s}^2}$$

4 Freely Falling Motion

17. **A tennis ball is thrown vertically upward with an initial speed of $17\,\text{m/s}$ and caught at the same level above the ground.**

    (a) How high does the ball rise?

    (b) How long was the ball in the air?

    (c) How long does it take to reach its highest point?

Solution: Take up as the positive direction and the throwing point to be the origin, so $y_0 = 0$.

(a) The ball goes so up until its vertical velocity becomes zero. For this part of ascending motion, we can write the free fall kinematic equation $v^2 - v_0^2 = -2g(y - y_0)$. Substituting the known values into it and solving for y, we get

$$v^2 - v_0^2 = -2g(y - y_0)$$

$$0 - 17^2 = -2(10)(y_{max} - 0)$$

$$\Rightarrow \boxed{y_{max} = 14.45\,\mathrm{m}}$$

(b) In all free fall practice problems, the best way to find the total flight time that the object was in the air is to use the kinematic equation $y - y_0 = -\frac{1}{2}gt^2 + v_0 t$. Then, substitute the coordinate of where the object landed on the ground into it.

As a rule of thumb, if the object returns back to the same point of the launch, its displacement vector is always zero, so $y - y_0 = 0$. Therefore, we will have

$$y - y_0 = -\frac{1}{2}gt^2 + v_0 t$$

$$0 = -\frac{1}{2}(10)t^2 + 17t$$

$$\Rightarrow \boxed{5t^2 - 17t = 0}$$

The expression obtained in the last step, can be solved by factoring out the time and setting the remaining to zero.

$$5t^2 - 17t = 0$$

$$t(5t - 17) = 0$$

$$\Rightarrow t = 0 \quad , \quad t = 3.4\,\mathrm{s}$$

The first result corresponds to the initial time, and the other time, $\boxed{t_{tot} = 3.4\,\mathrm{s}}$, is the amount of time the ball is in the air until it reaches the ground.

(c) At the highest point the vertical velocity is always zero, $v = 0$. Using the equation $v = v_0 - gt$, and solving for t, we will get

$$v = v_0 - gt$$
$$0 = 17 - (10)t$$
$$\Rightarrow t_{top} = 1.7\,\mathrm{s}$$

As you can see, the duration of ball's going up, in the absence of the air resistance, is always half the total flight time.

$$t_{top} = \frac{1}{2}t_{tot}$$

18. **From a height of $45\,\mathrm{m/s}$, a ball is dropped directly downward with an initial speed of $6\,\mathrm{m/s}$. How many seconds later does it strike the ground?**

Solution: Take up as the positive direction and the dropping point as the origin, so the initial height becomes $y_0 = 0$. The ball is moving downward so we must choose a sign for its initial velocity because velocity is a vector quantity in physics. Hence, in this case, the correct input for the initial velocity in the freely falling kinematic equations is $v_0 = -6\,\mathrm{m/s}$.

The ball strikes the ground $45\,\mathrm{m}$ below the chosen origin, so its correct coordinate is $y = -45\,\mathrm{m}$.

The only kinematic equation that relates all these variables to the time is $y - y_0 = -\frac{1}{2}gt^2 + v_0 t$. Substituting the numerical values into this equation, yields

$$y - y_0 = -\frac{1}{2}gt^2 + v_0 t$$

$$-45 - 0 = -\frac{1}{2}(10)t^2 + (-6)t$$

$$\Rightarrow \boxed{5t^2 + 6t - 45 = 0}$$

In the last step, after rearranging, we arrived at a quadratic equation, like $at^2 + bt + c = 0$, that its solution is found using the below formula

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a, b, c are some constants. In this case, we have

$$a = 5, b = 5, c = -45$$

Substituting the values into the above formula, we will have

$$t = \frac{-5 \pm \sqrt{5^2 - 4(5)(-45)}}{2(5)}$$

$$t = 2.46\,\mathrm{s} \quad , \quad \mathrm{t} = -3.66$$

Keep in mind that in all free fall practice problems, we must choose the positive time. Therefore, the ball would reach the ground about $2.5\,\mathrm{s}$ after dropping.

19. **From a $25 - \mathrm{m}$ building, a ball is thrown vertically upward at an initial velocity $20\,\mathrm{m/s}$. How long will it take to hit the ground?**

Solution: Origin is considered to be at the throwing point. Applying the position kinematic equation below to find the required time

$$\Delta y = -\frac{1}{2}\,gt^2 + v_0\,t$$
$$-25 = -\frac{1}{2}\,(10)\,t^2 + 20\,t$$

Rearranging and converting it into the standard form of quadratic equation $ax^2 + b\,x + c = 0$ as $t^2 - 4t - 5 = 0$, solutions are obtained as

$$\begin{aligned} x_{1,2} &= \frac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2a} \\ &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-5)}}{2(1)} \\ &= -1 \text{ and } 5 \end{aligned}$$

Therefore, the time needed the ball hit the ground is $5\,\text{s}$.

20. **From the top of a building with a height of $60\,\text{m}$, a rock is thrown directly upward at an initial velocity of $20\,\text{m/s}$. What is the rock's velocity at the instant of hitting the ground?**

Solution: Apply the time-independent kinematic equation as

$$\begin{aligned} v^2 - v_0^2 &= -2\,g\,\Delta y \\ v^2 - (20)^2 &= -2(10)(-60) \\ v^2 &= 1600 \\ \Rightarrow v &= 40\,\text{m/s} \end{aligned}$$

Therefore, the rock's velocity when it hit the ground is $v = -40\,\text{m/s}$.

5 Projectile motion

21. **A person kicks a ball with an initial velocity $15\,\text{m/s}$ at an angle 37° above the horizontal (neglect the air resistance). Find**

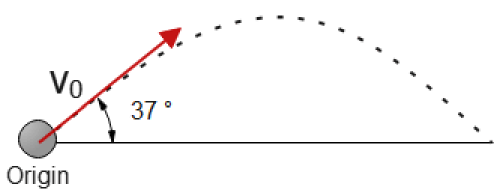
(a) the total time the ball is in the air.

(b) the horizontal distance traveled by the ball

Solution: To solve any projectile problems, first of all, adopt a coordinate system and draw its projectile path and put the initial and final positions, and velocities.

By doing this, you will be able to solve the relevant projectile equations easily.

Therefore, we choose the origin of the coordinate system to be at the throwing point, $x_0 = 0, y_0 = 0$.



(a) Here, the time between throwing and striking the ground is wanted.

In effect, the projectiles have two independent motions, one is in the horizontal direction with uniform motion at constant velocity i.e. $a_x = 0$, and the other is in the vertical direction under the effect of gravity with $a_y = -g$.

The kinematic equations that describe the horizontal and vertical distances are as follows

$$x = x_0 + (\underbrace{v_0 \cos\theta}_{v_{0x}})t$$

$$y = -\frac{1}{2}gt^2 + (\underbrace{v_0 \sin\theta}_{v_{0y}})t + y_0$$

By substituting the coordinates of the initial and final points into the vertical equation, we can find the total time the ball is in the air.

Setting $y = 0$ in the second equation, we have

$$y = -\frac{1}{2}gt^2 + (v_0 \sin\theta)t + y_0$$

$$0 = -\frac{1}{2}(9.8)t^2 + (15)\sin 37^\circ\, t + 0$$

By rearranging the above expression, we can get two solutions for t:

$$t_1 = 0$$

$$t_2 = \frac{2 \times 15 \sin 37^\circ}{9.8} = 1.84\,\mathrm{s}$$

The first time is for the starting moment and the second is the total time the ball was in the air.

(b) As mentioned above, the projectile motion is made up of two independent motions with different positions, velocities, and accelerations which two distinct kinematic equations describe those motions.

Between any two desired points in the projectile path, the time needed to move horizontally to reach a specific point is the same time needed to fall vertically to that point.

This is an important observation in solving projectile motion problems.

Therefore, time is the only common quantity in the horizontal and vertical motions of a projectile. In this problem, the time obtained in part (a) can be substituted in the horizontal kinematic equation, to find the distance traveled as below

$$\begin{aligned} x &= x_0 + (v_0 \cos\theta)t \\ &= 0 + (15)\cos 37^\circ\,(1.84) \\ &= 22.08 \quad \mathrm{m} \end{aligned}$$

6 Kinematics Equations

22. **An object moves the distance of $45\,\mathrm{m}$ in the time interval $5\,\mathrm{s}$ with a initial velocity and acceleration of v_0 and $2\,\mathrm{m/s^2}$, respectively. What is the initial velocity v_0?**

Solution: Known: $\Delta x = 45\,\text{m}$, $\Delta t = 5\,\text{s}$, $a = 2\,\text{m/s}^2$, $v_0 =?$. Use the following kinematic equation to find the unknown initial velocity

$$\begin{aligned}\Delta x &= \frac{1}{2}\,at^2 + v_0 t\\ 45 &= \frac{1}{2}(2)(5)^2 + v_0(5)\\ \Rightarrow &\boxed{v_0 = 4\,\text{m/s}}\end{aligned}$$

23. **An object, without change in direction, travels a distance of $50\,\text{m}$ with an initial speed $5\,\text{m/s}$ in $4\,\text{s}$. Find the object's velocity at the end of the given time interval.**

Solution: Known: $\Delta x = 50\,\text{m}$, $v_i = 5\,\text{m/s}$, $\Delta t = 4\,\text{s}$, $v_f =?$ With the above known values, we only use the following displacement kinematic equation to first find the acceleration

$$\begin{aligned}\Delta x &= \frac{1}{2}\,at^2 + v_i\,t\\ 50 &= \frac{1}{2}(a)(4)^2 + (5)(4)\\ \Rightarrow a &= \frac{30}{8} = \frac{15}{4}\end{aligned}$$

Now apply the below kinematic formula to find the final velocity

$$\begin{aligned}v_f &= v_i + a\,t\\ &= 5 + \frac{15}{4} \times 4 = 20\,\text{m/s}\end{aligned}$$

Alternative solution:Since in this problem we have two unknown that is acceleration and final velocity and the motion is constant acceleration, so one can use the below total displacement formula

$$\begin{aligned}\Delta x &= \frac{v_i + v_f}{2} \times \Delta t\\ 50 &= \frac{5 + v_f}{2} \times (4)\\ \Rightarrow v_f &= 20\,\text{m/s}\end{aligned}$$

24. **A car starts its motion from rest with a constant acceleration of $4\,\text{m/s}^2$. What is the average velocity of the car in the first $5\,\text{s}$ of the motion?**

Solution: Recall that once you have initial and final velocities of a moving object during a constant acceleration motion, then you can use $\bar{v} = \frac{v_i + v_f}{2}$ to find the average acceleration. In this problem, $v_i = 0$ and final velocity is obtained as

$$\begin{aligned}v_f &= v_0 + a\,t\\ &= 0 + (4)(5) = 20\,\text{m/s}\end{aligned}$$

Now use the above formula to find the average velocity as

$$\begin{aligned}\bar{v} &= \frac{0 + 20}{2}\\ &= 10\,\text{m/s}\end{aligned}$$

25. **A particle moves from rest with a uniform acceleration and travels $40\,\mathrm{m}$ in $4\,\mathrm{s}$. At what distance from the origin is this particle at the instant of $t = 10\,\mathrm{s}$?**

Solution: Known: $\Delta x = 40\,\mathrm{m}$, $\Delta t_1 = t-1-t_0 = 4\,\mathrm{s}$,$\Delta t_2 = t-2-t_0 = 10\,\mathrm{s}$ First, use the displacement kinematic equation to find the acceleration as

$$\begin{aligned}\Delta x &= \frac{1}{2}a\,t^2 + v_0 t\\ 40 &= \frac{1}{2}(a)(4)^2 + 0\\ \Rightarrow a &= 5\,\mathrm{m/s^2}\end{aligned}$$

Now use again that formula to find the displacement at the moment $t = 10\,\mathrm{s}$.

$$\begin{aligned}\Delta x &= \frac{1}{2}a\,t^2 + v_0 t\\ &= \frac{1}{2}(5)(10)^2 + 0\\ &= 250\,\mathrm{m}\end{aligned}$$

7 Circular Motion

26. **An $5-\mathrm{kg}$ object moves around a circular track of a radius of $18\,\mathrm{cm}$ with a constant speed of $6\,\mathrm{m/s}$. Find**

   (a) The magnitude and direction of the acceleration of the object.

   (b) The net force acting upon the object causing this acceleration.

Solution: When an object moves around a circular path at a constant speed, the only acceleration that experiences is the centripetal acceleration or radial acceleration.

This kind of acceleration is always toward the center of the circle and its magnitude is found by the following formula

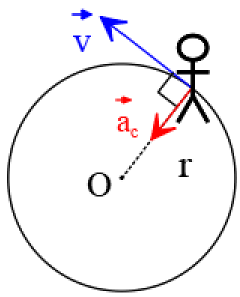
$$a_c = \frac{v^2}{r}$$

where v is the constant speed with which the object revolves the circle, and r is the radius of the circle.

(a) The track is circular and the speed of the object is constant, so a centripetal acceleration directed toward the center is applied to the object whose magnitude is as follows

$$a_c = \frac{6^2}{0.18\,\mathrm{m}} = 50\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

In the figure below, a top view of the motion is sketched.



(b) By applying Newton's second law along the direction of the centripetal acceleration, we can find the magnitude of the net force causing the acceleration as follows

$$F_{net} = m\frac{v^2}{r}$$

Therefore,

$$F_{net} = 5 \times 50 = 250 \quad \text{N}$$

Note: At each point along the circular path, the instantaneous velocity of the revolving object is tangent to the path. The direction of this velocity changes, but its magnitude remains constant.

27. **In a merry-go-round moves with a speed of 3 m/s, a 25-kg child sits 3 m from its center. Calculate**

   (a) The centripetal acceleration of the child

   (b) The net horizontal force acted upon the child

   (c) Compare the above force with the child's weight

   Solution: (a) The child has a circular motion with a centripetal acceleration as $a_c = v^2/r$ where v is the constant speed of the revolving object. Therefore,

$$a_c = \frac{3^2}{3} = 3 \quad \frac{\text{m}}{\text{s}^2}$$

   (b) The net force is found using Newton's second law as $F_{net} = ma_c$ which yields

$$F_{net} = 25 \times 3 = 75 \quad \text{N}$$

   This force is in the same direction as the acceleration, toward the center of the circle.

   (c) Weight is mass times the gravitational acceleration at that place (g) or $w = mg$. The ratio between these two forces is

$$\frac{F_{net}}{w} = \frac{75}{25 \times 10} = 0.3$$

8 Friction Force

28. **A constant force of $10\,\mathrm{N}$ is applied to a $2-\mathrm{kg}$ crate on a rough surface that is sitting on it. The crate undergoes a frictional force against the force that moves it over the surface.**

   (a) Assuming the coefficient of friction is $\mu_k = 0.24$, find the magnitude of the friction force that opposes the motion.

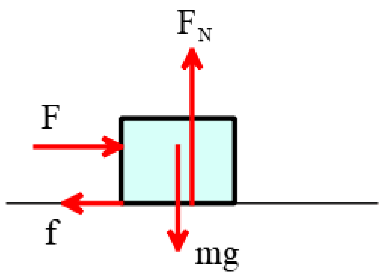
   (b) What is the net force on the crate?

   (c) What acceleration does the crate obtain?

Solution: The kinetic friction force is the force that opposes the motion of a moving object and its magnitude is given by the formula below $f_k = \mu_k F_N$, where μ_k is the coefficient of kinetic friction, and F_N is the normal force on the object due to contact with a surface.

In all problems involving the coefficient of friction, if the normal force is not given, then you must apply Newton's second law in the vertical direction to find it.

(a) The crate does not move vertically or lift off the surface, so the forces in this direction must be balanced with each other. The free-body diagram below shows that two forces are acting on the crate: an upward normal force F_N, and a downward weight force $W = mg$.



Thus,

$$F_N = mg = 2 \times 10 = 20\,\mathrm{N}$$

Now that the normal force is known, we can use kinetic friction force formula $f_k = \mu_k F_N$, to find its magnitude

$$f_k = \mu_k F_N = 0.24 \times 20 = 4.8\,\mathrm{N}$$

(b) " Net force" means the sum vector of forces. In the horizontal direction, two forces act on the crate: external force F, and kinetic friction force f_k. These two forces apply in the opposite direction.

The subtraction of these two forces gives us the net (resultant) force on the crate. So,

$$F_{net} = F - f_k = 10 - 4.8 = 5.2\,\mathrm{N}$$

(c) According to Newton's second law of motion, if a force of F is applied to a body of mass m, then it undergoes an acceleration whose magnitude is given by $a = \frac{F}{m}$. So, the acceleration that this crate experiences is found as

$$a = \frac{F_{net}}{m} = \frac{4.8}{2} = 2.4\,\mathrm{m/s^2}$$

8.1 static friction

29. **A** $100\,\mathrm{N}$ **is applied to a** $50\,\mathrm{kg}$ **box setting on a surface. Suppose the coefficient of static friction is** $\mu_s = 0.25$**. Is this applied force enough to move the box?**

    Solution: As mentioned earlier, for static friction, we can only find its maximum value, in contrast to kinetic friction. At any other moment, before reaching the maximum value, this is the external force that determines the magnitude of the static friction.

    If externally applied force on an object is less than the maximum of the static friction $F < f_{s,max}$, then the static friction equals the magnitude of the external force, $f_s = F$.

    In this case, first find the maximum value of the static friction

    $$f_{s,max} = \mu_s F_N = 0.25 \times 500 = 125\,\mathrm{N}$$

    where we substituted mg for the normal force since the object is on a horizontal surface and a horizontal force is applied to it.

    As you can see, the applied external force is not enough to move the object since $F < f_{s,max}$. So, the magnitude of the static friction is $f_s = 100\,\mathrm{N}$.

30. **A** $300\,\mathrm{N}$ **is required to start a box to move over a rough level floor. If the coefficient of static friction between them is** 0.35**, find the mass of the box?**

    Solution: The box is initially at rest, and a force of $300\,\mathrm{N}$ puts it on the verge of motion. So, this external force must be enough to overcome the maximum force of static friction, $F = f_{s,max} = \mu_s F_N$.

    The object does not move in the vertical direction, so the net force vertically must be zero,

    $$F_N - mg = 0 \Rightarrow F_N = mg$$

    Substituting this into the above expression and solving for m, we will have

    $$F = \mu_s F_N = \mu_s(mg)$$

    $$\Rightarrow m = \frac{F}{\mu_s g}$$

    $$= \frac{300}{0.35 \times 10}$$

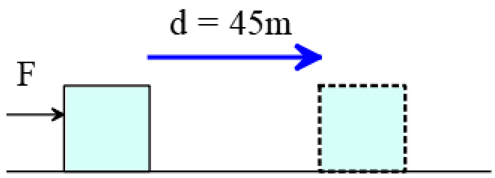
    $$= 85.7 \quad \mathrm{kg}$$

9 Work

9.1 Constant Force

31. **A constant force of** $1200\,\mathrm{N}$ **is required to push a car along a straight line. A person displaces the car** $45\,\mathrm{m}$**. How much work is done by the person?**

Solution: If a constant force F acts on an object over a distance of d, and F is parallel to d, then the work done by force F is the product of the force times distance.



In this case, a force of $1200\,\mathrm{N}$ displaces the car $45\,\mathrm{m}$. The pushing force is parallel to the displacement. So, the work done by the person is equal to

$$W = Fd = 1200 \times 45 = 54000\,\mathrm{J}$$

The SI unit of work is the joule, J.

32. **You lift a book of mass $2\,\mathrm{kg}$ at constant speed straight upward a distance of $2\,\mathrm{m}$. How much work is done during this lifting by you?**

Solution: The force you apply to lift the book must be balanced with the book's weight. So, the exerted force on the book is

$$F = mg = 2 \times 10 = 20 \quad \mathrm{N}$$

The book is lifted 2 m vertically. The force and displacement are both parallel to the up, so the work done by the person is the product of them.

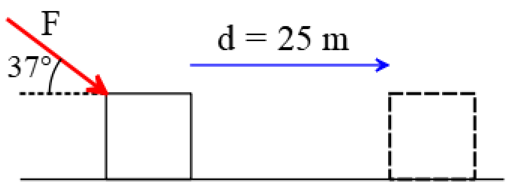
$$W = Fd = 20 \times 2 = 40 \quad \mathrm{J}$$

33. **A force of $F = 20\,\mathrm{N}$ at an angle of 37° is applied to a $3-\mathrm{kg}$ object initially at rest. The object has displaced a distance of $25\,\mathrm{m}$ over a frictionless horizontal table. Determine the work done by**

(a) The applied force

(b) The normal force exerted by the table

(c) The force of gravity



Solution: In this problem, the force makes an angle with the displacement. In such cases, we should use the formula for work in physics, $W = Fd\cos\theta$ where θ is the angle between force

F and displacement. To this object an external force F, normal force F_N, and gravity of force $w = mg$ are applied.

(a) Using vector decompositions, the component of the force parallel to the displacement is found to be $F_{\parallel} = F\cos\theta$. Thus, the product of this component parallel to the displacement times the magnitude of displacement gives us the work done by external force F as below

$$\begin{aligned} W_F &= \underbrace{F\cos\theta}_{F_{\parallel}}\, d \\ &= (20 \times \cos 37^\circ)(25) \\ &= 400 \quad \text{J} \end{aligned}$$

(b) Now, we want to find the work done by the normal force. But, let's define what is the normal force.

In physics, "Normal" means perpendicular. When an object is in contact with a surface, a contact force is exerted on the object. The component of the contact force perpendicular to the surface is called the normal force.

Thus, by definition, the normal force is always perpendicular to the displacement. So, the angle between F_N and displacement d is 90°. Hence, the work done by the normal force is determined to be

$$W_N = F_N d\cos\theta = (30)(25)\cos 90^\circ = 0$$

(c) The weight of the object is the same as the force of gravity. This force applies to the object vertically downward and the displacement of the object is horizontal. So, again the angle between these two vectors is $\theta = 90^\circ$. Hence, the work done by the force of gravity is zero.

9.2 Work in Circular Motions

9.3 Work on an incline

10 Work-Kinetic Energy Theorem

11 Momentum in One-Dimension

12 Conservation of Momentum

13 Impulse and Momentum

13.1 Impulse and Force-time graphs

14 Sound

15 Elastic Collision in One Dimension

16 Heat and Temperature

17 Latent heat of Vaporization

18 Latent Heat of Fusion

19 Simple Pendulum

20 Simple Harmonic Motion (SHM)

21 Standing Waves

22 Coulomb's Law

34. **Two like and equal charges are at a distance of $d = 5\,\text{cm}$ and exert a force of $F = 9 \times 10^{-3}\,\text{N}$ on each other.**

    (a) **Find the magnitude of each charge?**

    (b) **What is the direction of the electrostatic force between them?**

Solution: The magnitude of the force between two rest point charges q and q' separated by a distance d is given by Coulomb's law as below

$$F = k\,\frac{|q|\,|q'|}{d^2}$$

where $k \approx 8.99 \times 10^9\,\frac{\mathrm{N.m^2}}{\mathrm{C^2}}$ is the Coulomb constant and the magnitudes of charges denoted by $|\cdots|$.

Let the magnitude of charges be $|q_1| = |q_2| = |q|$, Now by substituting the known numerical values of F and distance d, and solving for $|q|$ we get

$$\begin{aligned} F &= k\,\frac{|q_1|\,|q_2|}{d^2} \\ 9 \times 10^{-3} &= (8.99 \times 10^9)\frac{|q|^2}{(0.05)^2} \\ \Rightarrow q^2 &= 25 \times 10^{-16} \\ \Rightarrow q &= 5 \times 10^{-8}\,\mathrm{C} \end{aligned}$$

In the second equality, we converted the distance from cm to m to coincide with SI units.

The direction of the Coulomb force depends on the sign of the charges. Two like charges repel and two unlike ones attract each other.

Since q_1 and q_2 have the same signs so the electric force between them is repulsive.

35. **A point charge of $q = 4\,\mu\mathrm{C}$ is $3\,\mathrm{cm}$ apart from the charge $q' = 1\,\mu\mathrm{C}$.**

    (a) Find the magnitude of the Coulomb force that one particle exerts on the other.

    (b) Is the force attractive or repulsive?

Solution: Known values:

$$\begin{aligned} |q| &= 4\,\mu\mathrm{C} \\ |q^{'}| &= 1\,\mu\mathrm{C} \\ d = 3\,\mathrm{cm} &= 3 \times 10^{-2}\,\mathrm{m} \end{aligned}$$

(a) Coulomb's law gives the magnitude of the electric force between two stationary (motionless) point charges so by applying it we have

$$\begin{aligned} F &= k\,\frac{|q|\,|q'|}{d^2} \\ &= (8.99 \times 10^9)\,\frac{(4 \times 10^{-6})(1 \times 10^{-6})}{(0.03)^2} \\ &= 40\,\mathrm{N} \end{aligned}$$

(b) Since the charges have opposite signs so the electric force between them is attractive.

23 Electric Field

24 Electric Flux

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