The force exerted by a point charge q_1 on another point charge q_2 located at a distance r away is given by the following formula

$$\vec{F} = k\frac{|q_1 q_2|}{r^2}\hat{r}$$

where $\hat{r}$ is a unit vector points from q_1 toward q_2.
Note that Coulomb's law gets only the magnitude of the electric force between two point charges.

These questions are intended for the college level and are difficult. For simple and more relevant practice problems on Coulomb's law for the high school level, refer to here.

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1 Coulomb's Law: Problems and Solutions

1. **Compute the electric force between two charges of $5\times 10^{-9}\,\mathrm{C}$ and $-3\times 10^{-8}\,\mathrm{C}$ which are separated by $d = 10\,\mathrm{cm}$.**

   Solution: the magnitude of the electrostatic force between two point charges is given by Coulomb's law as

$$\begin{aligned} F &= k\frac{|q_1 q_2|}{d^2} \\ &= \left(9\times 10^9\right)\frac{|(5\times 10^{-9})(-3\times 10^{-8})|}{(0.1)^2} \\ &= 135\times 10^{-6} \quad \mathrm{N} \end{aligned}$$

   where $|\cdots|$ denote the magnitude of the charges.

   Note that in Coulomb's law force formula, the sign of charges is not included only its absolute values must be entered.

2. **Two spheres located at distance of $d = 5\,\mathrm{cm}$ attract one another with a force of $F = 3\,\mathrm{mN}$. If one of them has three times more charges than the other, find the electric force between them?**

   Solution: let one of charges be $q_1 =?$ and the other $q_2 = 3q_1$. Then using Coulomb's law formula and solving for the unknown charges, we have

$$F = k\frac{|q_1 q_2|}{d^2}$$

$$3 = \left(9\times 10^9\right)\frac{|q_1 \times 3q_1|}{0.05}$$

$$\Rightarrow q_1^2 = \frac{(3\times 10^{-3})(0.05)}{(9\times 10^9 \times 3)}$$

$$= 0.5\times 10^{-14}$$

Taking square root from both sides gives

$$q_1 = 0.75 \times 10^{-7}\,\text{C}$$

Thus, the magnitude of the charges are $q_1 = 0.075\,\mu\text{C}$ and $q_2 = 0.225\,\mu\text{C}$.

3. **A point charge $q_1 = 2\,\mu\text{C}$ located at origin and another point charge $q_2 = -5\,\mu\text{C}$ is on the coordinate $(x = 3, y = 4)\,\text{m}$.**

   (a) Find the electric force on charge q_1.

   (b) Is the force attractive or repulsive?

   Solution: the distance between two point charges is found using distance formula (Pythagorean theorem) as below

   $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

   which gives

   $$d = \sqrt{3^2 + 4^2} = 5 \quad \text{m}$$

   (a) Now, use Coulomb's law formula to find the magnitude of the force between two point charges as below

   $$F = k\frac{|q_1 q_2|}{d^2}$$

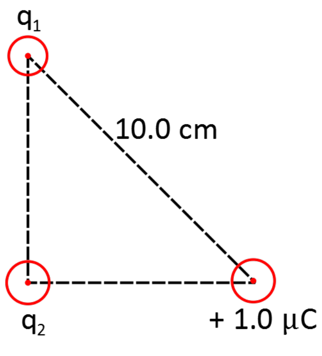
   $$= \left(9 \times 10^9\right) \frac{|(2 \times 10^{-6})(-5 \times 10^{-6})|}{5^2}$$

   $$= 3.6 \times 10^{-3} \quad \text{N}$$

   (b) Coulomb's law gives only the magnitude of the electric force. Being repulsive or attractive depends on the signs of charges. *Like charges attract and unlike charges repel each other.*

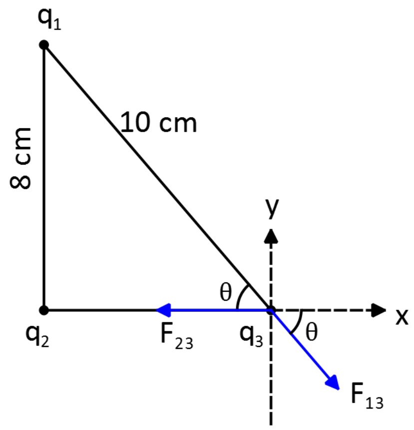
   Here, the two charges have opposite signs so the electric force between them is attractive.

4. **Three point charges are fixed in place in the right triangle shown below, in which $q_1 = 0.71\,\mu\text{C}$ and $q_2 = -0.67\,\mu\text{C}$. What is the magnitude and direction of the electric force on the $+1.0\,\mu\text{C}$ (let's call this q_3) charge due to the other two charges?**



Solution: First, find the electric force due to each charge on the q_3, then use the superposition principle to do the vector sum of them.

In the figure below, all forces on q_3 are sketched. Recall that the like charges repel each other and unlike charges attract.



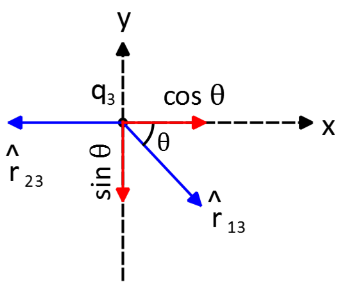
The magnitude of Coulomb's forces on the charge q_3 is obtained as below

$$\vec{F}_{13} = k\frac{|q_1|\,|q_3|}{r_{13}^2}\hat{r}_{13}$$

$$= \left(9\times 10^9\right)\frac{\left(0.71\times 10^{-6}\right)\left(1\times 10^{-6}\right)}{(0.1)^2}\left(\cos\theta\ \hat{x}+\sin\theta(-\hat{y})\right)$$

$$= 0.639 \quad \text{N}$$

Where $\hat{r}_{13}$ is the unit vector (a vector whose length is unity) along the line connecting the two charges and decomposed as shown in the figure.



Since $q_1 > 0$ so the electric field lines are along the line between q_1 and q_3 and directed away from q_3. From the geometry we see that $\sin\theta = \frac{8}{10}$ and $\cos\theta = \frac{\sqrt{10^2-8^2}}{10} = \frac{6}{10}$. Therefore,

$$\vec{F}_{13} = 0.639\,(0.6\,\hat{x} + 0.8\,(-\hat{y}))$$
$$= (0.383\hat{x} - 0.511\hat{y})\ \text{N}$$

Now find the electric force due to the q_2 on q_3 i.e. $\vec{F}_{23}$

$$\vec{F}_{23} = k\frac{|q_2|\,|q_3|}{r_{23}^2}\hat{r}_{23}$$

$$= (9\times 10^9)\frac{\left|-0.67\times 10^{-6}\right|\left(1\times 10^{-6}\right)}{\left(10^2-8^2\right)\times 10^{-4}\,\mathrm{m}^2}(-\hat{y})$$

$$= (-1.675\ \hat{y})\quad \mathrm{N}$$

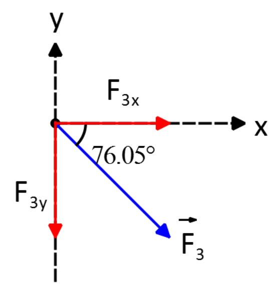
Therefore, the resultant force on the q_3 is

$$\begin{aligned}\vec{F}_3 &= \vec{F}_{13} + \vec{F}_{23}\\ &= (0.383\hat{x} - 0.511\hat{y}) + (-1.675\,\hat{y})\\ &= (0.383\,\hat{x} - 2.186\,\hat{y})\quad \mathrm{N}\end{aligned}$$

The direction of the net force with the x axis are determined by $\tan\alpha\ = |F_y|\,/\,|F_x|$, so

$$\alpha = \tan^{-1}\left(\frac{2.058}{0.511}\right)\ = 76.05^\circ$$

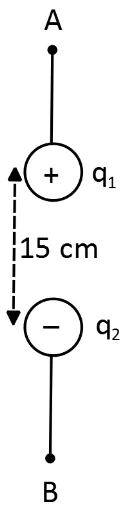
Since $F_{3x} > 0$ and $F_{3y} < 0$, the net force lies in the fourth quadrant.



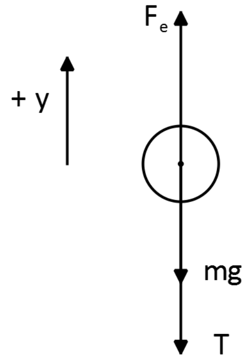
Using Pythagorean theorem, its magnitude is also found to be

$$\left|\vec{F}_3\right| = \sqrt{(0.511)^2 + (-2.058)^2} = 2.12\ \mathrm{N}$$

5. **Two small insulating spheres are attached to silk threads and aligned vertically as shown in the figure. These spheres have equal masses of $40\,\mathrm{g}$, and carry charges q_1 and q_2 of equal magnitude $2.0\,\mu\mathrm{C}$ but opposite sign.The spheres are brought into the positions shown in the figure, with a vertical separation of $15\,\mathrm{cm}$ between them. Note that you cannot neglect gravity. What is the tension in the lower threads?**



Solution: There are three forces acting on q_2. The attractive electrostatic force F_e due to q_1, tension force in the thread, and gravity. Thus, its free body diagram is as follows



The system is in equilibrium so the net force on the q_2 is zero i.e.

$$(\Sigma F_y)_2 = 0$$

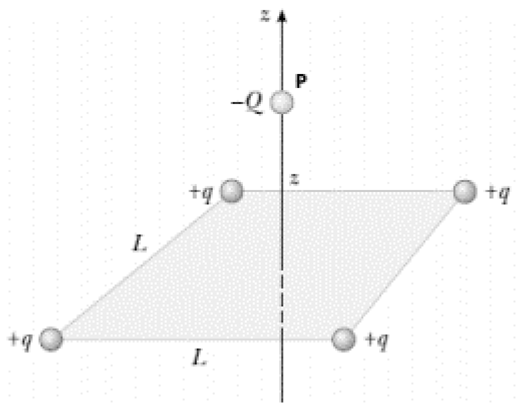
$$\Rightarrow F_e - T - mg = 0$$

$$\Rightarrow T = \frac{k\,|q_1|\,|q_2|}{(15)^2} - mg$$

$$\Rightarrow T = 9 \times 10^9 \frac{\left(2 \times 10^{-6}\right)\left(2 \times 10^{-6}\right)}{(0.15)^2} - (0.040 \times 9.8) = 1.208 \text{ N}$$

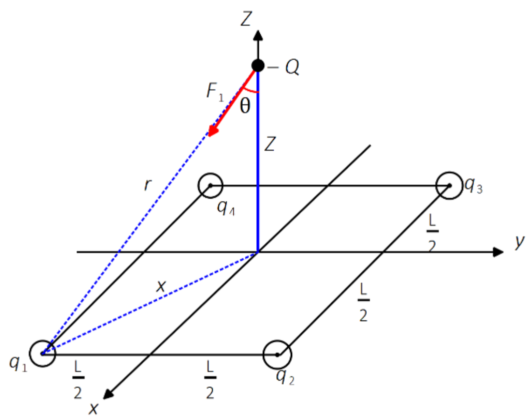
6. **Four identical particles, each having charge $+q$, are fixed at the corners of a square of side L. A fifth point charge $-Q$ (at P point) lies a distance z along the line**

perpendicular to the plane of the square and passing through the center of the square. Determine the force exerted by the other four charges on $-Q$.



Solution: Because the magnitude and distance of all charges are equal so consider

$$|F_1| = |F_2| = |F_3| = |F_4| = k\frac{|qQ|}{r^2}$$



By symmetry consideration, $F_x = F_y = 0$. So the direction of one of the forces is:

$$\vec{F}_{1z} = k\frac{|qQ|}{r^2}\cos\theta\,\left(-\hat{k}\right) = -k\frac{|qQ|}{r^3}z\hat{k}$$

Where we have used from the geometry of the problem $\cos\theta\ = z/r$. By symmetry $\vec{F}_{1z} = \vec{F}_{2z} = \vec{F}_{3z} = \vec{F}_{4z} = -k\frac{|qQ|}{r^3}z\hat{k}$ So

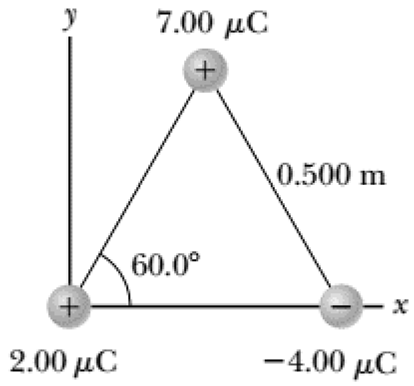
$$\vec{F}_z = \Sigma_{i=1}^{4}\vec{F}_{iz} = -4k\frac{qQ}{r^3}z\hat{k}$$

In terms of the parameters of the square and using the Pythagorean theorem, we have:

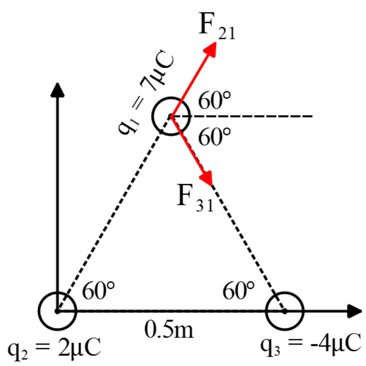
$$x = \frac{\sqrt{2}}{2} L \ , \ r = \sqrt{z^2 + \left(\frac{\sqrt{2}}{2} L\right)^2}$$

$$\vec{F}_z = -4k \frac{Qq}{\left(z^2 + \left(\frac{\sqrt{2}}{2} L\right)^2\right)^{\frac{3}{2}}} z \, \hat{k}$$

7. **Three point charges are located at the corners of an equilateral triangle an in the figure. Find the magnitude and direction of the net electric force on the $7\,\mu$C charge.**



Solution: Same as the previous problem, first we must calculate each of the electric forces due to the $2\,\mu$C, $-4\,$C charges exerted on the third charge then use the superposition principle to determine the net electric force on it.



$$\begin{aligned}\vec{F}_{21} &= k\frac{|q_1q_2|}{r_{12}^2}\hat{r}_{21}\\ &= 9\times10^9\frac{2\times10^{-6}\times7\times10^{-6}}{(0.5)^2}\hat{r}_{21}\\ &= 0.504\ \hat{r}_{21}\,\mathrm{N}\end{aligned}$$

$\hat{r}_{21}$ is the unit vector points from q_2 toward q_1 so if one decomposes it, we get

$$\vec{F}_{21} = 0.504\left(\frac{1}{2}\hat{x}+\frac{\sqrt{3}}{2}\hat{y}\right)\mathrm{N}$$

(Notation: F_{12} is the force exerted by point charge $q_1$on point charge q_2)

$$\begin{aligned}\vec{F}_{31} &= k\frac{|q_1q_3|}{r_{13}^2}\hat{r}_{31}\\ &= 9\times10^9\frac{\left|7\times10^{-6}\times(-4)\times10^{-6}\right|}{(0.5)^2}\left(\cos60^\circ\ \hat{x}+\sin60^\circ\ (-\hat{y})\right)\ \mathrm{N}\\ &= 1.008\ \left(\frac{1}{2}\hat{x}+\frac{\sqrt{3}}{2}(-\hat{y})\right)\ \mathrm{N}\end{aligned}$$

Using superposition principle: $\vec{\mathrm{F}}_1$=$\vec{\mathrm{F}}_{31}$+$\vec{\mathrm{F}}_{32}$, we obtain

$$\begin{aligned}\vec{\mathrm{F}}_1 &= 0.504\left(\frac{1}{2}\hat{x}+\frac{\sqrt{3}}{2}\hat{y}\right)+1.008\ \left(\frac{1}{2}\hat{x}+\frac{\sqrt{3}}{2}(-\hat{y})\right)\\ &= 0.756\ \hat{x}-0.437\ \hat{y}\ \ (\mathrm{N})\end{aligned}$$

And its magnitude is

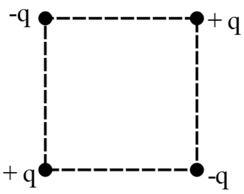
$$\left|\vec{F}_1\right| = \sqrt{(0.756)^2+(-0.437)^2} = 0.873\,\mathrm{N}$$

And also the direction of the resultant force with the horizontal axis (x) is

$$\alpha = \tan^{-1}\left(\frac{|-0.437|}{|0.756|}\right) = 30.02^\circ$$

Since $F_{1x}>0$ and $F_{1y}<0$ so the net force lies in the fourth quadrant.

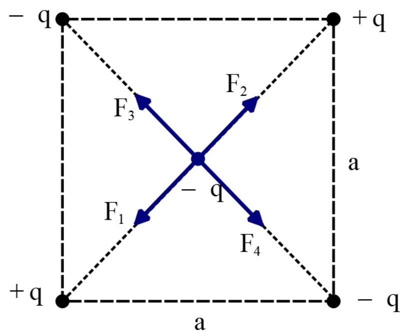
8. **Four point charges are at the corners of a square. The distance from each corner to the center is** 0.3 m. **At the center, there is a** $-q$ **point charge. What is the magnitude of the net force on this charge?**



Solution: Note: the electric force vector between two point charges located at distance r from each other is $\vec{F} = k\frac{|q_1 q_2|}{r^2}\hat{r}$.

When there is a system of point charges and we want to find the net force on one of the charges, we must use the superposition principle i.e. the vector sum of the individual electric forces on the desired charge: $\vec{\mathrm{F}}_{\mathrm{net}}=\vec{\mathrm{F}}_1+\vec{\mathrm{F}}_2+\vec{\mathrm{F}}_3+\dots$ So we must vector sum the individual forces due to four point charges on the $-q$ in the center.

The drawing below shows the direction of the individual forces.



$$|F_1| = |F_2| = k\frac{|(-q)\,(+q)|}{(0.3)^2} = +\frac{kq^2}{0.09}$$

$$|F_3| = |F_4| = k\frac{|(-q)\,(-q)|}{(0.3)^2} = +k\frac{q^2}{0.09}$$

Because the F_1, F_2 and F_3, F_4 are separately in opposite directions to each other (i.e. $\vec{F}_1 = -\vec{F}_2$ and $\vec{F}_3 = -\vec{F}_4$) so the net force is

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$$

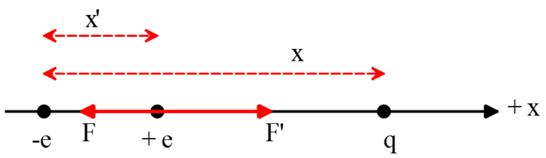
9. **A electron is fixed at the position $x = 0$, and a second charge q is fixed at $x = 4 \times 10^{-9}$ m (to the right). A proton is now placed between the two at $x' = 1 \times 10^{-9}$ m. What must the charge q be (magnitude and sign) so that the proton is in equilibrium?**

Solution: The magnitude of the electric force between two point charges q and q' located at distance r from each other is given by the Coulomb's law as follows

$$F = k\frac{|q\,q'|}{r^2}$$

Where $k = 9 \times 10^9\ \mathrm{N \cdot m^2/C^2}$. The directions of forces the two charges exert on each other are always along the line joining them.

The figure below is a free-body diagram for the proton. Let us consider the charge q to be positive. In such a case, F is the force exerted on the proton by the electron, and F' is the force



exerted by the charge q on it. Now compute these forces and use the superposition principle to find the total force acting on the proton.

$$\vec{F}_{tot} = \vec{F} + \vec{F}' = k\frac{|(-e)\,(+e)|}{x'^2}\left(-\hat{i}\right) + k\frac{|(+e)\,q|}{(x-x')^2}\left(+\hat{i}\right)$$

Since the charge q is in equilibrium state so the total force exerted on it must be zero

$$\vec{F}_{tot} = 0\ ,\ \text{equilibrium condition}$$

$$k\frac{|(-e)\,(+e)|}{x'^2}\left(-\hat{i}\right) + k\frac{|(+e)\,q|}{(x-x')^2}\left(+\hat{i}\right) = 0$$

$$\Rightarrow\ \frac{e}{x'^2} = \frac{|q|}{(x-x')^2} \Rightarrow e\left(x-x'\right)^2 = |q|\,x'^2$$

$$e(4\ \text{nm} - 1\text{nm})^2 = |q|\,(1\text{nm})^2$$

$$\Rightarrow\ |q| = 9e$$

If we assume that the charge q is negative, we get the same result.

10. **Four point charges lie on the corners of a square of side $L = a\sqrt{2}$. What is the magnitude of the net Coulomb force at the place of charge $-q$?**

Solution: Similar to the previous problem, first find (using the definition of Coulomb's law) each electric force on charge $-q$ then form the vector sum of them and determine its magnitude. Charges q_1 and q_3 (in the figure below) have the same magnitude and are at equal distances from q_4 so the magnitude of their Coulomb forces acted on q_4 are equal.

$$\text{Coulomb's law}:\ F = k\,\frac{|q|\,|q'|}{d^2}$$

$$F_{14} = F_{34} = k\,\frac{|q|\,|-q|}{a\sqrt{2}}$$

$$= k\,\frac{|q|^2}{2a^2}$$

Since the two charges have the opposite signs so the electric force (attractive) on q_4 due to q_1 is to the left and due to q_3 is downward as shown in the figure. Therefore, the net force of them, F, is $\sqrt{2}\,F_{14}$ or $\sqrt{2}\,F_{34}$.

Similarly, find the electrostatic force F_{24} due to charge q_2 on q_4.

$$F_{24} = k\,\frac{|q|\,|-q|}{(2a)^2}$$

$$= \frac{1}{4}\,\frac{k\,|q|^2}{a^2}$$

The distance between q_2 and q_4 is the diagonal length of the square which is obtained using the Pythagorean theorem.

The charges q_2 and q_4 have opposite signs, so the Coulomb force between them is attractive and directed inward along the diagonal of the square.

The magnitude of the net Coulomb force on q_2 is determined by adding the other magnitudes since they are directed in the same direction along the diagonal of the square. Thus,

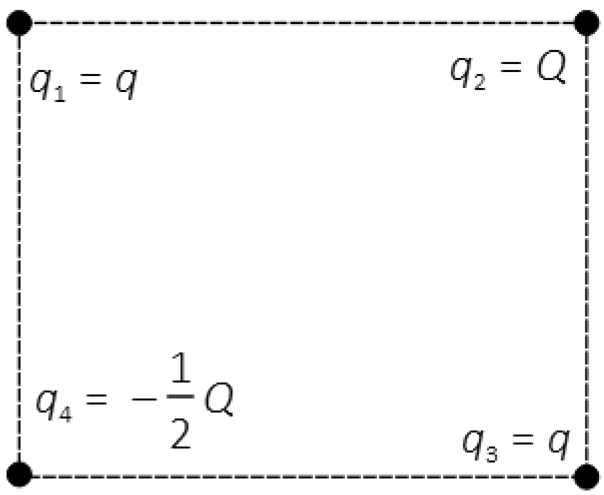
$$F_2 = F + F_{24}$$

$$= k\,\frac{|q|^2}{2a^2} + \frac{1}{4}\,\frac{k\,|q|^2}{a^2}$$

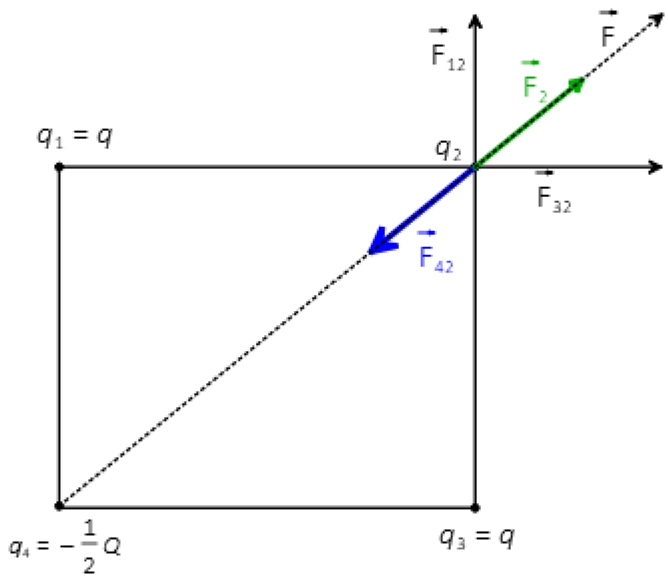
$$= k\,\frac{|q|^2}{a^2}\left(\frac{1}{2} + \frac{1}{4}\right)$$

$$= \frac{3}{4}\,k\,\frac{|q|^2}{a^2}$$

11. **Four point charges are located on the corners of a square shown in the figure. If the net Coulomb force on q_2 is zero, what is the ratio of $\frac{Q}{q}$?**



Solution: Since $|q_1| = |q_3| = q$ and placed at a equal distance of charge q_2 so $F_{12} = F_{32}$. We know that the resultant vector of two perpendicular and equal vectors F is given as $\sqrt{2}\,F$ so, in this case, the magnitude of the net force acting on charge q_2 due to q_1 and q_3 is $F = \sqrt{2}\,F_{12}$ along the diagonal $(q_2 - q_4)$ of the square and directed outward as shown in the figure.



The total electric force on charge q_2 is the vector sum (superposition principle) of $\vec{F_2} = \vec{F} + \vec{F}_{42}$ since said that it is zero $\vec{F_2} = 0$ so the electrostatic force of q_4 on q_2 i.e. $\vec{F}_{42}$ must be equal in magnitude and opposite in direction with $\vec{F}$. Therefore, by equating the magnitudes of the forces i.e. $F = F_{42}$ we obtain

$$F = F_{42}$$

$$\sqrt{2}\,F_{12} = F_{42}$$

$$\sqrt{2}\,k\,\frac{|q_1|\,|q_2|}{a^2} = k\,\frac{|q_4|\,|q_2|}{(\sqrt{2}\,a)^2}$$

$$\sqrt{2}\frac{|q|\,|Q|}{1} = \frac{|\frac{1}{2}Q|\,|Q|}{2}$$

$$\Rightarrow \frac{Q}{q} = 4\sqrt{2}$$

12. **In The configuration of three point charges, as shown in the figure below, the Coulomb force on each charge is zero. Determine the ratio of charges q_3 and q_2 i.e. $\frac{q_3}{q_2}$.**

Solution: Since the ratio of the $\frac{q_3}{q_2}$ is required and the net force on each charge is zero we must balance the forces on the charge q_1 because in this case, the magnitude of q_1 cancels from both sides as below,

$$F_{21} = F_{31}$$

$$k\,\frac{|q_1|\,|q_2|}{(20)^2} = k\,\frac{|q_1|\,|q_3|}{(30)^2}$$

$$\frac{|q_2|}{400} = \frac{|q_3|}{900}$$

$$\Rightarrow \frac{|q_3|}{|q_2|} = \frac{9}{4}$$

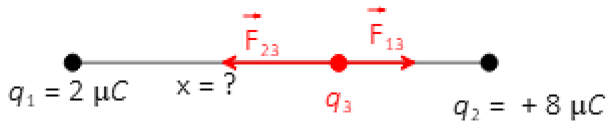
Note: since the expression above is an equality so no need to convert the units to SI.

Now that the ratio of the magnitudes of the charges is obtained we must determine its signs. As you can see in the figure because the forces $\vec{F}_{31}$ and $\vec{F}_{32}$ are in the opposite directions (to produce a zero net force on q_1) so the charges q_2 and q_3 must be unlike.

The exact sign of charges can not be determined as long as at least the sign of one charge is given. See the later problem.

13. **Two point charges $q_1 = +2\,\mu\text{C}$ and $q_2 = +8\,\mu\text{C}$ are $30\,\text{cm}$ apart from each other. Another charge q is placed so that the three charges are brought to a balance. What is the magnitude and sign of the charge q?**

Solution: To find the location of the third charge, place a positive (or negative) test charge q_3 somewhere between q_1 and q_2. Since all charges here are positive (negative), by Coulomb's law, the electrostatic forces on the test charge are repulsive (attractive) and to the left (right) and right (left) of it. Consequently, the net electric force can be zero between them at a distance of say x from charge q_1.



Now, balance the magnitude of the forces on the test charge q_3 as below to find the location of it

$$F_{13} = F_{23}$$

$$k\,\frac{|q_1|\,|q_3|}{x^2} = k\,\frac{|q_2|\,|q_3|}{(30-x)^2}$$

$$\frac{2}{x^2} = \frac{8}{(30-x)^2}$$

$$\Rightarrow 2x = 30 - x$$

$$\Rightarrow x = 10\,\text{cm}$$

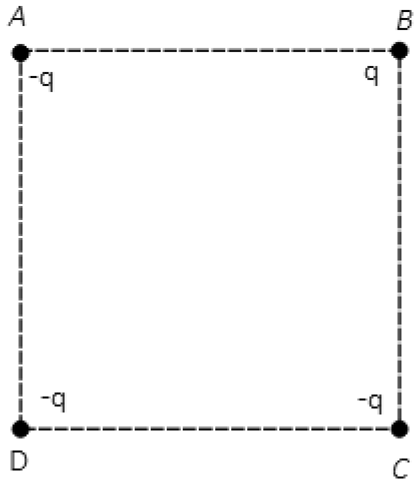
In above, the required charge q_3 is canceled from both sides and one can not find its sign and value. To find the magnitude and sign of q_3, balance the forces on another charge, say q_1 as below

$$F_{31} = F_{21}$$
$$k\,\frac{|q_1|\,|q_3|}{(10)^2} = k\,\frac{|q_2|\,|q_1|}{(30)^2}$$
$$\frac{|q_3|}{100} = \frac{8}{900}$$
$$\Rightarrow |q_3| = \frac{8}{9}$$

The electric force $\vec{F}_{21}$ is repulsive and directed to the $-x$ axis. Since the net force on each charge is zero the charge q_3 must be negative to provide an attraction force in the opposite direction of $\vec{F}_{21}$ that is to the $+x$ axis.

Therefore, the third charge is negative, located at a distance of 10 cm between the two other charges.

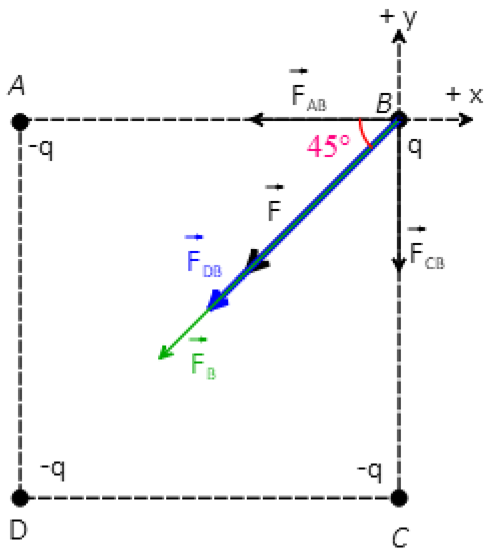
14. **In the corners of a square of side L, four point charges are fixed as shown in the figure below. What angle does make the net Coulomb force vector on the charge q located at the point B in the upper right corner with the horizontal?**



Solution: In this problem, there is no need to do any explicit calculation, only justify the desired direction.

The electric force vector on the charge q at the corner B is the vector sum of the forces acting by the other charges $-q$ on it. Therefore, using superposition principle, we have

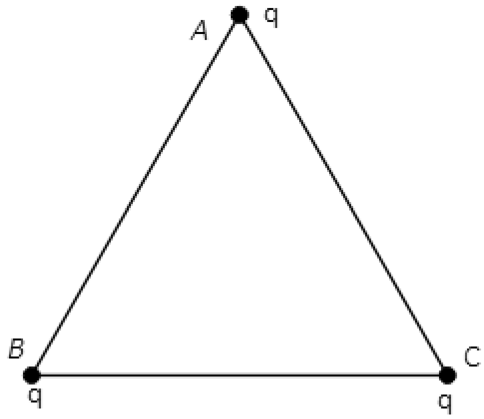
$$\vec{F}_B = \vec{F}_{AB} + \vec{F}_{DB} + \vec{F}_{CB}$$



Similar to the previous problems, since the magnitude and distance of charges located at A and C are equal and the same so $|\vec{F}_{AB}| = |\vec{F}_{CB}| = F$. On the other hand, those forces are attractive and directed to the points A and C as shown in the figure. Thus, their resultant electric force lies along the diagonal of BD points inward with the magnitude of $\sqrt{2}\,F$.

The force between charge $-q$ at point D and q at point B is also attractive, lies along the diagonal of BD, and points inward. Therefore, Adding these three force vectors gives a resultant Coulomb force vector $\vec{F}_B$ directed with an angle of $(180 + 45)^\circ$ along the BD diagonal as shown in the figure.

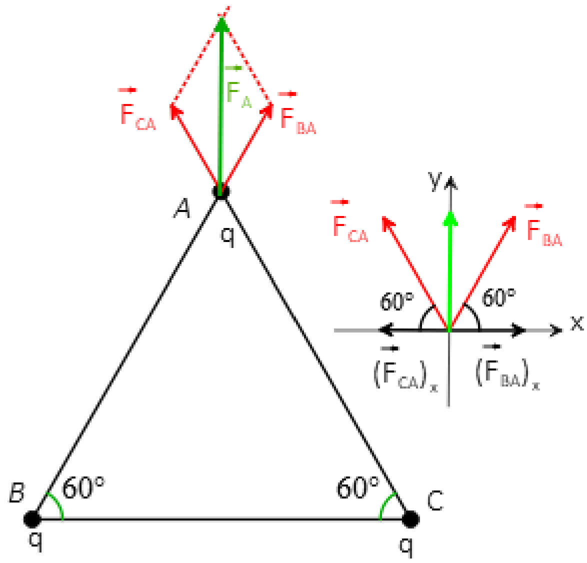
15. **Three equal point charges are placed at the vertices of an equilateral triangle of side a. What is the magnitude and direction of the Coulomb force on the charge q at the point A? ($q = 10\,\mu\text{C}$ and $a = \sqrt[4]{3}\,\text{m}$).**



Solution: first find the magnitudes of $\vec{F}_{BA}$ and $\vec{F}_{CA}$ using Coulomb's force law as below

$$\begin{aligned} F_{BA} &= k\,\frac{|q_B|\,|q_A|}{d^2} \\ &= k\,\frac{q^2}{\left(\sqrt[4]{3}\right)^2} \\ &= k\,\frac{q^2}{\sqrt{3}} \\ &= (9\times 10^9)\,\frac{(10\times 10^{-6})^2}{\sqrt{3}} \\ &= \frac{9}{\sqrt{3}}\times 10^{-1}\,\text{N} \end{aligned}$$

Since the distance to q_A and the magnitudes of q_B and q_C are the same so $F_{BA} = F_{CA} = F$.



Now find the direction of the electrostatic forces above using vector components. $\vec{F}_{BA}$ makes an angle of 60° with the $+x$ direction and $\vec{F}_{CA}$ an angle of 60° with the $-x$ direction. Thus, the above forces can be written in the following vector form

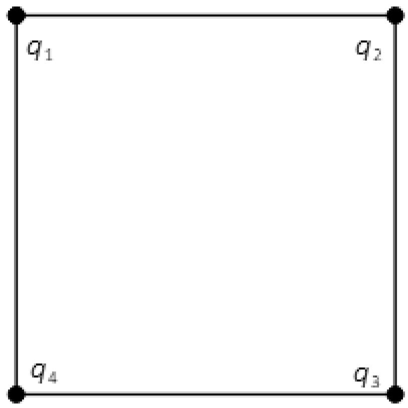
$$\vec{F}_{BA} = \underbrace{|\vec{F}_{BA}|}_{F}\left(\cos 60^\circ\,\hat{i} + \sin 60^\circ\,\hat{j}\right)$$
$$\vec{F}_{CA} = \underbrace{|\vec{F}_{CA}|}_{F}\left(\cos 60^\circ\,(-\hat{i}) + \sin 60^\circ\,\hat{j}\right)$$

The x-components will add up to zero which gives the x-component of the net force on the charge on the position A. The sum of the y-components also gives

$$\begin{aligned} F_{Ay} &= F\,\sin 60^\circ + F\,\sin 60^\circ \\ &= 2F\,\sin 60^\circ \\ &= 2F \times \left(\frac{\sqrt{3}}{2}\right) \\ &= \sqrt{3}\,F \\ &= \sqrt{3} \times \frac{9}{\sqrt{3}} \times 10^{-1} \\ &= 0.9\,\mathrm{N} \end{aligned}$$

Therefore, the resultant Coulomb force on q_A directed upward and is written as $\vec{F}_A = 0.9\,\hat{j}$.

16. **Four unknown point charges are held at the corners of a square. Suppose q_4 is at equilibrium and Let $q_1 = q_3 = -5\,\mu\text{C}$ then what is the magnitude of the charge q_2 and the sign of the ratio of $\frac{q_2}{q_4}$.**



Solution: Since q_4 is at equilibrium, the net electric force on it must be zero. Applying the superposition principle at point 4 we get

$$\vec{F}_{net-on-q_4} = 0$$
$$\vec{F}_{14} + \vec{F}_{24} + \vec{F}_{34} = 0$$
$$\Rightarrow \vec{F}_{14} + \vec{F}_{34} = -\vec{F}_{24}$$

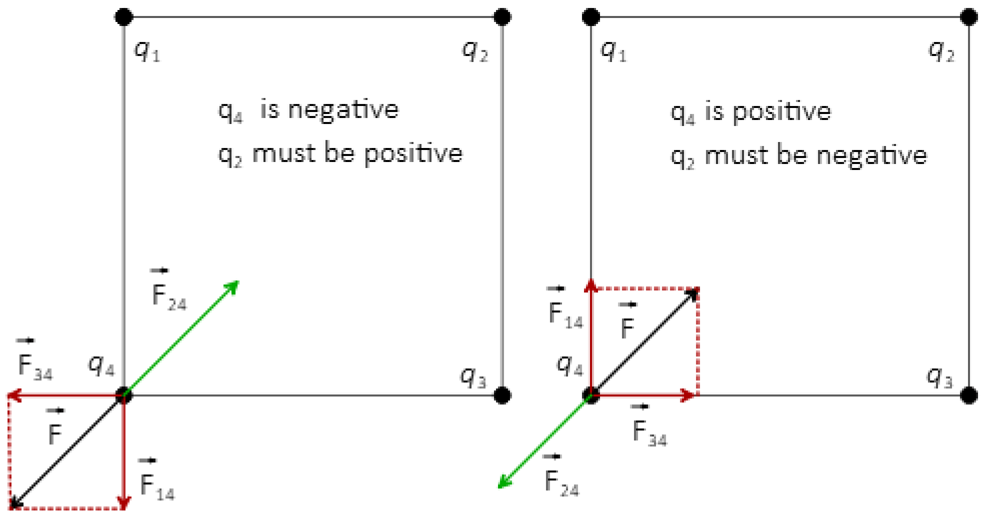
Let's consider first the charge q_4 is positive. Because of being negative of the charges q_1 and q_3, their forces on q_4 are attractive, to the right and up direction which gives a net force F along the diagonal of the square and directed inward.

In this case, the electric force $\vec{F}_{24}$ must be diagonally and directed outward to cancel the contribution F (See the right figure). This result tells us that the force between q_2 and q_4 must be repulsive, or they must have like charges.

Because we assumed $q_4 > 0$, so q_2 is also positive. Thus, $\frac{q_2}{q_4} > 0$ for this situation.

Similar reasoning can be also applied for the case of a negative q_4 charge (left figure). Consequently, q_4 and q_2 are unlike charges or its ratio is $\frac{q_2}{q_4} < 0$.

Consequently, this analysis tells us that q_2 must be always positive.



Since $|q_1| = |q_3| = |q|$ and are at an equal distance to q_4 so their forces on q_4 due to these charges are also equal with magnitude (using Coulomb's law formula)

$$\begin{aligned} F_{14} = F_{34} &= \\ &= k\frac{|q_1\, or\, q_3||q_4|}{a^2} \\ &= k\,\frac{|q|\,|q_4|}{a^2} \end{aligned}$$

Pythagorean theorem gives the net electric force on q_4 due to q_1 and q_3 as

$$F = \sqrt{F_{14}^2 + F_{34}^2}$$

$$= \sqrt{F_{14}^2 + F_{14}^2}$$

$$= \sqrt{2}\,F_{14}$$

Now we proceed to determine the magnitude of q_2 by applying the equilibrium condition on charge q_4 (the magnitude of the forces along the square diagonal (F and F_{24}) must be equal) as below

$$F = F_{24}$$

$$\sqrt{2}\, F_{14} = k\, \frac{|q_2|\,|q_4|}{(\sqrt{2}\, a)^2}$$

$$\sqrt{2}\, k\, \frac{|q_1|\,|q_4|}{a^2} = k\, \frac{|q_2|\,|q_4|}{(\sqrt{2}\, a)^2}$$

$$\sqrt{2}\, \frac{\left(5 \times 10^{-6}\right)}{a^2} = \frac{|q_2|}{2a^2}$$

$$\Rightarrow |q_2| = 10\sqrt{2}\, \mu\mathrm{C}$$

the first equality is the equilibrium condition. Therefore, the charge q_2 has a magnitude of $\boxed{10\sqrt{2}\, \mu\mathrm{C}}$.

17. **Two point charges of $q_1 = +2\,\mu\mathrm{C}$ and $q_2 = -8\,\mu\mathrm{C}$ are at a distance of $d = 10\,\mathrm{cm}$. Where must a third charge q_3 be placed so that the net Coulomb force acted upon it is zero?**

Solution: Put a positive (or negative) test charge q_3 between them and examine whether the net Coulomb force on it is zero or not. In this case, the net electrostatic force on the positive (negative) test charge due to the charges q_1 and q_2 is to the right (left). Thus, there is no space between them to balance a test charge.

Now place that test charge q_3 outside them, say in the left of the charge q_1 at a distance x from it. One can see that, in this case, the forces on the q_3 can be balanced and canceled by each other. Therefore, apply Coulomb's force law and find the unknown x as below,

$$\begin{aligned}
F_{13} &= F_{23} \\
k\, \frac{|q_1|\,|q_3|}{x^2} &= k\, \frac{|q_2|\,|q_3|}{(10+x)^2} \\
\frac{|2 \times 10^{-6}|}{x^2} &= \frac{|-8 \times 10^{-6}|}{(10+x)^2} \\
\frac{1}{x^2} &= \frac{4}{(10+x)^2} \\
\Rightarrow \frac{1}{4} &= \frac{x^2}{(x+10)^2} \\
\Rightarrow \frac{x}{x+10} &= \pm\frac{1}{2}
\end{aligned}$$

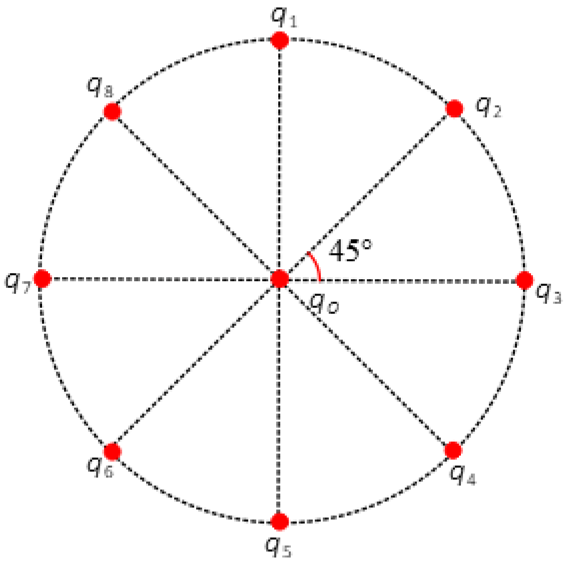
In the fifth equality, the square root is taken from both sides. Solving the last equation for x, we get $x = 10\,\mathrm{cm}$.

18. **In the figure below, what is the magnitude and direction of the net Coulomb force vector acted on the charge $q_O = q$ by the eight other charges placed on the**

circumference of a circle of radius $R = 100\,\text{cm}$. Let $q_0 = +20\,\mu\text{C}$ and other charges be

$$q_1 = q_2 = q_3 = q_4 = q_5 = q_7 = q_8 = q = 50\,\mu\text{C}$$
$$q_6 = -q$$

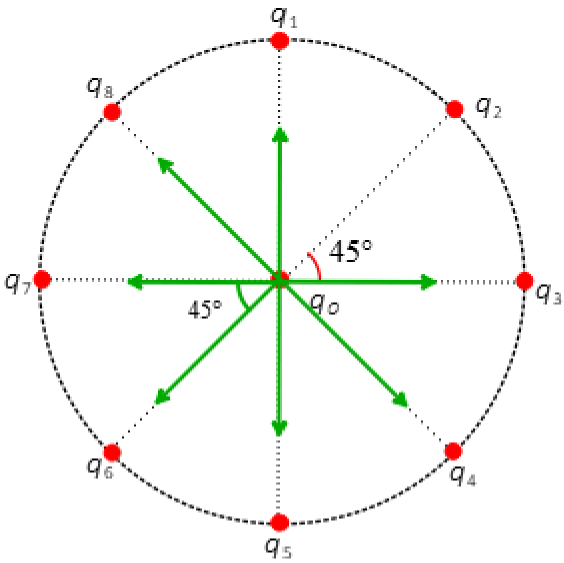
The charge q_0 is held at the center of circle.



Solution: Using the symmetry of the charge configuration, one can realize that the electric forces due to a pair of charges (q_1, q_5), (q_2, q_8) and (q_3, q_7) on the charge at the origin q_O are equal in magnitude and opposite in direction, so cancel each other. Consequently, the net force on the charge q at the center is only due to the charges q_6 and q_2 which its magnitudes (F_{1O} and F_{6O}) are computed by applying Coulomb's law as below

$$\begin{aligned}
F &= F_{1O} = F_{6O} \\
F &= k\,\frac{|q_1|\,|q|}{R^2} = k\,\frac{|q_6|\,|q|}{R^2} \\
F &= k\,\frac{|q|\,|q|}{R^2} = k\,\frac{|-q|\,|q|}{R^2} \\
\Rightarrow F &= k\,\frac{|q|^2}{R^2} \\
&= (9\times 10^9)\,\frac{(50\times 10^{-6})(20\times 10^{-6})}{(100\times 10^{-2})^2} \\
&= 9\,\text{N}
\end{aligned}$$

The charge q_6 attracts and q_1 repels the charge q at the center so the magnitude of the net electric force at point O is 2 times the magnitude of the force between q_6 or q_1 and q at center i.e. $|\vec{F}_O| = 2F = 19\,\text{N}$.



The resultant electric force $\vec{F}_O$ lies on the third quadrant, points radially outward, and makes an angle of $(180+45)^\circ$ with the positive x axis or 45° with the $-x$ axis. Its vector form is written as follows

$$\vec{F}_O = 18\left(\cos 45^\circ\,(-\hat{i}) + \sin 45^\circ\,(-\hat{j})\right)$$

More Problems about all topics of electrostatic are also provided here.