The force exerted by a point charge q_1 on another point charge q_2 located at a distance r away is given by the following formula

$$\vec{F} = k\frac{|q_1 q_2|}{r^2}\hat{r}$$

where $\hat{r}$ is a unit vector points from q_1 toward q_2.
Note that Coulomb's law gets only the magnitude of the electric force between two point charges.

These questions are intended for the college level and are difficult. For simple and more relevant practice problems on Coulomb's law for the high school level, refer to here.

1 Coulomb's Law: Problems and Solutions

1. **Compute the electric force between two charges of $5\times10^{-9}\,\mathrm{C}$ and $-3\times10^{-8}\,\mathrm{C}$ which are separated by $d = 10\,\mathrm{cm}$.**

   Solution: the magnitude of the electrostatic force between two point charges is given by Coulomb's law as

$$\begin{aligned} F &= k\frac{|q_1 q_2|}{d^2} \\ &= \left(9\times10^9\right)\frac{|(5\times10^{-9})(-3\times10^{-8})|}{(0.1)^2} \\ &= 135\times10^{-6} \quad \mathrm{N} \end{aligned}$$

   where $|\cdots|$ denote the magnitude of the charges.

   Note that in Coulomb's law force formula, the sign of charges is not included only its absolute values must be entered.

2. **Two spheres located at distance of $d = 5\,\mathrm{cm}$ attract one another with a force of $F = 3\,\mathrm{mN}$. If one of them has three times more charges than the other, find the electric force between them?**

   Solution: let one of charges be $q_1 =?$ and the other $q_2 = 3q_1$. Then using Coulomb's law formula and solving for the unknown charges, we have

$$F = k\frac{|q_1 q_2|}{d^2}$$

$$3 = \left(9\times10^9\right)\frac{|q_1\times 3q_1|}{0.05}$$

$$\Rightarrow q_1^2 = \frac{(3\times10^{-3})(0.05)}{(9\times10^9\times3)}$$

$$= 0.5\times10^{-14}$$

Taking square root from both sides gives

$$q_1 = 0.75 \times 10^{-7}\,\mathrm{C}$$

Thus, the magnitude of the charges are $q_1 = 0.075\,\mu\mathrm{C}$ and $q_2 = 0.225\,\mu\mathrm{C}$.

3. **A point charge $q_1 = 2\,\mu\mathrm{C}$ located at origin and another point charge $q_2 = -5\,\mu\mathrm{C}$ is on the coordinate $(x = 3, y = 4)\,\mathrm{m}$.**

   (a) Find the electric force on charge q_1.

   (b) Is the force attractive or repulsive?

   Solution: the distance between two point charges is found using distance formula (Pythagorean theorem) as below

   $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

   which gives

   $$d = \sqrt{3^2 + 4^2} = 5 \quad \mathrm{m}$$

   (a) Now, use Coulomb's law formula to find the magnitude of the force between two point charges as below

   $$F = k\frac{|q_1 q_2|}{d^2}$$

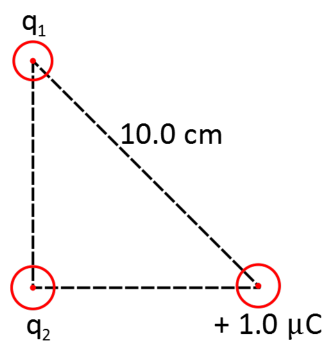
   $$= \left(9 \times 10^9\right) \frac{|(2 \times 10^{-6})(-5 \times 10^{-6})|}{5^2}$$

   $$= 3.6 \times 10^{-3} \quad \mathrm{N}$$

   (b) Coulomb's law gives only the magnitude of the electric force. Being repulsive or attractive depends on the signs of charges. *Like charges attract and unlike charges repel each other.*

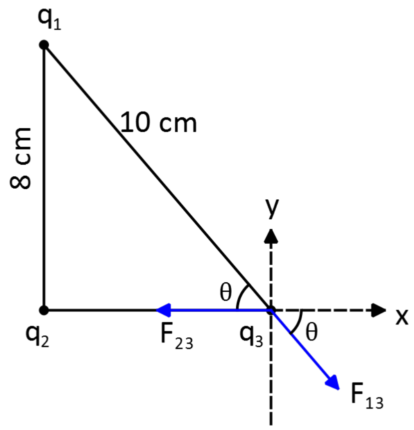
   Here, the two charges have opposite signs so the electric force between them is attractive.

4. **Three point charges are fixed in place in the right triangle shown below, in which $q_1 = 0.71\,\mu\mathrm{C}$ and $q_2 = -0.67\,\mu\mathrm{C}$. What is the magnitude and direction of the electric force on the $+1.0\,\mu\mathrm{C}$ (let's call this q_3) charge due to the other two charges?**



Solution: First, find the electric force due to each charge on the q_3, then use the superposition principle to do the vector sum of them.

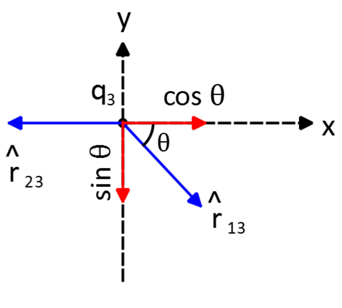
In the figure below, all forces on q_3 are sketched. Recall that the like charges repel each other and unlike charges attract.



The magnitude of Coulomb's forces on the charge q_3 is obtained as below

$$\begin{aligned}\vec{F}_{13} &= k\frac{|q_1|\,|q_3|}{r_{13}^2}\hat{r}_{13}\\ &= \left(9\times 10^9\right)\frac{\left(0.71\times 10^{-6}\right)\left(1\times 10^{-6}\right)}{(0.1)^2}\left(\cos\theta\ \hat{x}+\sin\theta(-\hat{y})\right)\\ &= 0.639 \quad \text{N}\end{aligned}$$

Where $\hat{r}_{13}$ is the unit vector (a vector whose length is unity) along the line connecting the two charges and decomposed as shown in the figure.



Since $q_1 > 0$ so the electric field lines are along the line between q_1 and q_3 and directed away from q_3. From the geometry we see that $\sin\theta\ = \frac{8}{10}$ and $\cos\theta\ = \frac{\sqrt{10^2-8^2}}{10} = \frac{6}{10}$. Therefore,

$$\begin{aligned}\vec{F}_{13} &= 0.639\,(0.6\,\hat{x}+0.8\,(-\hat{y}))\\ &= (0.383\hat{x}-0.511\hat{y})\ \text{N}\end{aligned}$$

Now find the electric force due to the q_2 on q_3 i.e. $\vec{F}_{23}$

$$\vec{F}_{23} = k\frac{|q_2|\,|q_3|}{r_{23}^2}\hat{r}_{23}$$

$$= (9\times 10^9)\frac{\left|-0.67\times 10^{-6}\right|\left(1\times 10^{-6}\right)}{\left(10^2-8^2\right)\times 10^{-4}\,\mathrm{m}^2}(-\hat{y})$$

$$= (-1.675\ \hat{y}) \quad \mathrm{N}$$

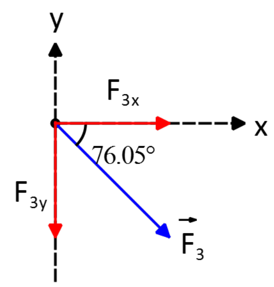
Therefore, the resultant force on the q_3 is

$$\begin{aligned}\vec{F}_3 &= \vec{F}_{13} + \vec{F}_{23}\\ &= (0.383\hat{x} - 0.511\hat{y}) + (-1.675\,\hat{y})\\ &= (0.383\,\hat{x} - 2.186\,\hat{y}) \quad \mathrm{N}\end{aligned}$$

The direction of the net force with the x axis are determined by $\tan\alpha\ = |F_y|\,/\,|F_x|$, so

$$\alpha = \tan^{-1}\left(\frac{2.058}{0.511}\right) \ = 76.05^\circ$$

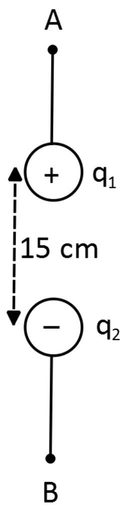
Since $F_{3x} > 0$ and $F_{3y} < 0$, the net force lies in the fourth quadrant.



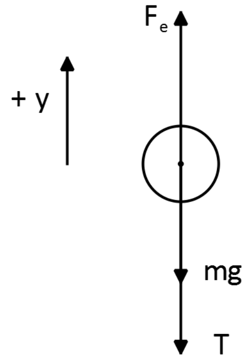
Using Pythagorean theorem, its magnitude is also found to be

$$\left|\vec{F}_3\right| = \sqrt{(0.511)^2 + (-2.058)^2} = 2.12\ \mathrm{N}$$

5. **Two small insulating spheres are attached to silk threads and aligned vertically as shown in the figure. These spheres have equal masses of $40\,\mathrm{g}$, and carry charges q_1 and q_2 of equal magnitude $2.0\,\mu\mathrm{C}$ but opposite sign.The spheres are brought into the positions shown in the figure, with a vertical separation of $15\,\mathrm{cm}$ between them. Note that you cannot neglect gravity. What is the tension in the lower threads?**



Solution: There are three forces acting on q_2. The attractive electrostatic force F_e due to q_1, tension force in the thread, and gravity. Thus, its free body diagram is as follows



The system is in equilibrium so the net force on the q_2 is zero i.e.

$$(\Sigma F_y)_2 = 0$$

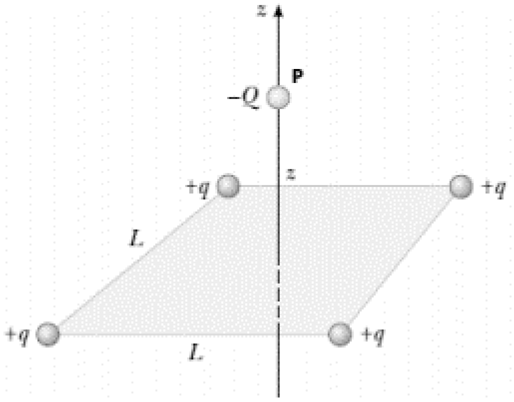
$$\Rightarrow F_e - T - mg = 0$$

$$\Rightarrow T = \frac{k\,|q_1|\,|q_2|}{(15)^2} - mg$$

$$\Rightarrow T = 9 \times 10^9 \frac{\left(2 \times 10^{-6}\right)\left(2 \times 10^{-6}\right)}{(0.15)^2} - (0.040 \times 9.8) = 1.208 \text{ N}$$

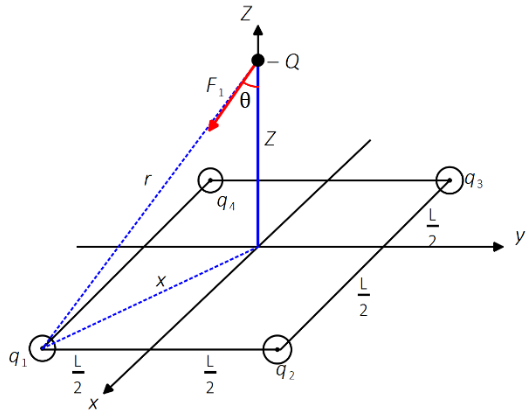
6. **Four identical particles, each having charge $+q$, are fixed at the corners of a square of side L. A fifth point charge $-Q$ (at P point) lies a distance z along the line**

perpendicular to the plane of the square and passing through the center of the square. Determine the force exerted by the other four charges on $-Q$.



Solution: Because the magnitude and distance of all charges are equal so consider

$$|F_1| = |F_2| = |F_3| = |F_4| = k\frac{|qQ|}{r^2}$$



By symmetry consideration, $F_x = F_y = 0$. So the direction of one of the forces is:

$$\vec{F}_{1z} = k\frac{|qQ|}{r^2}\cos\theta\,\left(-\hat{k}\right) = -k\frac{|qQ|}{r^3}z\hat{k}$$

Where we have used from the geometry of the problem $\cos\theta\ = z/r$. By symmetry $\vec{F}_{1z} = \vec{F}_{2z} = \vec{F}_{3z} = \vec{F}_{4z} = -k\frac{|qQ|}{r^3}z\hat{k}$ So

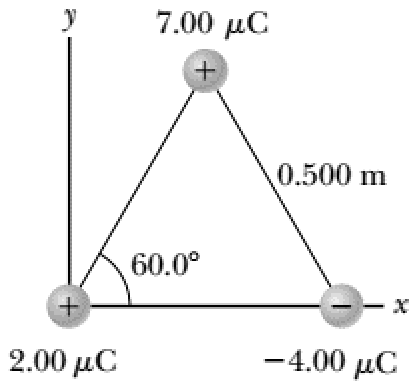
$$\vec{F}_z = \Sigma_{i=1}^{4}\vec{F}_{iz} = -4k\frac{qQ}{r^3}z\hat{k}$$

In terms of the parameters of the square and using the Pythagorean theorem, we have:

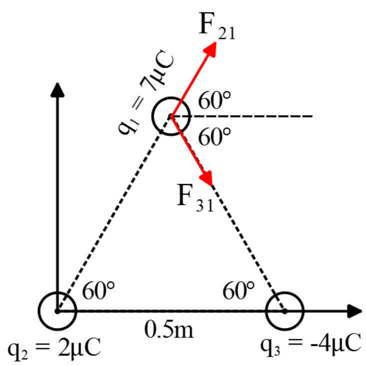
$$x = \frac{\sqrt{2}}{2}L \ , \ r = \sqrt{z^2 + \left(\frac{\sqrt{2}}{2}L\right)^2}$$

$$\vec{F}_z = -4k\frac{Qq}{\left(z^2 + \left(\frac{\sqrt{2}}{2}L\right)^2\right)^{\frac{3}{2}}} z\,\hat{k}$$

7. **Three point charges are located at the corners of an equilateral triangle an in the figure. Find the magnitude and direction of the net electric force on the $7\,\mu$C charge.**



Solution: Same as the previous problem, first we must calculate each of the electric forces due to the $2\,\mu$C, $-4\,$C charges exerted on the third charge then use the superposition principle to determine the net electric force on it.



$$\begin{aligned}\vec{F}_{21} &= k\frac{|q_1 q_2|}{r_{12}^2}\hat{r}_{21} \\ &= 9\times 10^9 \frac{2\times 10^{-6}\times 7\times 10^{-6}}{(0.5)^2}\hat{r}_{21} \\ &= 0.504\ \hat{r}_{21}\,\mathrm{N}\end{aligned}$$

$\hat{r}_{21}$ is the unit vector points from q_2 toward q_1 so if one decomposes it, we get

$$\vec{F}_{21} = 0.504\left(\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\right)\mathrm{N}$$

(Notation: F_{12} is the force exerted by point charge $q_1$on point charge q_2)

$$\begin{aligned}\vec{F}_{31} &= k\frac{|q_1 q_3|}{r_{13}^2}\hat{r}_{31} \\ &= 9\times 10^9 \frac{\left|7\times 10^{-6}\times(-4)\times 10^{-6}\right|}{(0.5)^2}\left(\cos 60^\circ\ \hat{x} + \sin 60^\circ\ \ (-\hat{y})\right)\ \mathrm{N} \\ &= 1.008\ \left(\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}(-\hat{y})\right)\ \mathrm{N}\end{aligned}$$

Using superposition principle: $\vec{\mathrm{F}}_1 = \vec{\mathrm{F}}_{31} + \vec{\mathrm{F}}_{32}$, we obtain

$$\begin{aligned}\vec{\mathrm{F}}_1 &= 0.504\left(\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\right) + 1.008\ \left(\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}(-\hat{y})\right) \\ &= 0.756\ \hat{x} - 0.437\ \hat{y}\ \ (\mathrm{N})\end{aligned}$$

And its magnitude is

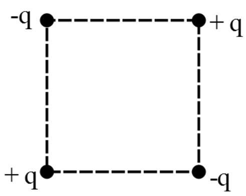
$$\left|\vec{F}_1\right| = \sqrt{(0.756)^2 + (-0.437)^2} = 0.873\,\mathrm{N}$$

And also the direction of the resultant force with the horizontal axis (x) is

$$\alpha = \tan^{-1}\left(\frac{|-0.437|}{|0.756|}\right) = 30.02^\circ$$

Since $F_{1x} > 0$ and $F_{1y} < 0$ so the net force lies in the fourth quadrant.

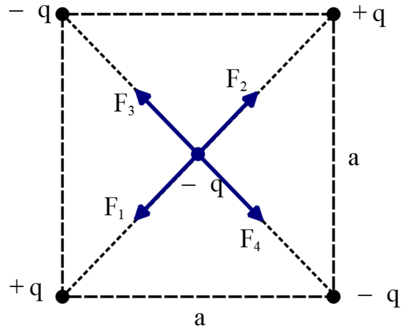
8. **Four point charges are at the corners of a square. The distance from each corner to the center is 0.3 m. At the center, there is a $-q$ point charge. What is the magnitude of the net force on this charge?**



Solution: Note: the electric force vector between two point charges located at distance r from each other is $\vec{F} = k\frac{|q_1 q_2|}{r^2}\hat{r}$.

When there is a system of point charges and we want to find the net force on one of the charges, we must use the superposition principle i.e. the vector sum of the individual electric forces on the desired charge: $\vec{\mathrm{F}}_{\mathrm{net}}=\vec{\mathrm{F}}_1+\vec{\mathrm{F}}_2+\vec{\mathrm{F}}_3+\dots$ So we must vector sum the individual forces due to four point charges on the $-q$ in the center.

The drawing below shows the direction of the individual forces.



$$|F_1| = |F_2| = k\frac{|(-q)\,(+q)|}{(0.3)^2} = +\frac{kq^2}{0.09}$$

$$|F_3| = |F_4| = k\frac{|(-q)\,(-q)|}{(0.3)^2} = +k\frac{q^2}{0.09}$$

Because the F_1, F_2 and F_3, F_4 are separately in opposite directions to each other (i.e. $\vec{F_1} = -\vec{F_2}$ and $\vec{F_3} = -\vec{F_4}$) so the net force is

$$\vec{F}_{net} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} = 0$$

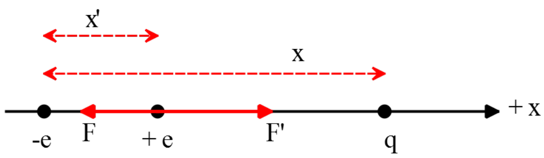
9. **A electron is fixed at the position $x = 0$, and a second charge q is fixed at $x = 4 \times 10^{-9}$ m (to the right). A proton is now placed between the two at $x' = 1 \times 10^{-9}$ m. What must the charge q be (magnitude and sign) so that the proton is in equilibrium?**

Solution: The magnitude of the electric force between two point charges q and q' located at distance r from each other is given by the Coulomb's law as follows

$$F = k\frac{|q\,q'|}{r^2}$$

Where $k = 9 \times 10^9\ \mathrm{N \cdot m^2/C^2}$. The directions of forces the two charges exert on each other are always along the line joining them.

The figure below is a free-body diagram for the proton. Let us consider the charge q to be positive. In such a case, F is the force exerted on the proton by the electron, and F' is the force



exerted by the charge q on it. Now compute these forces and use the superposition principle to find the total force acting on the proton.

$$\vec{F}_{tot} = \vec{F} + \vec{F}' = k\frac{|(-e)\,(+e)|}{x'^2}\left(-\hat{i}\right) + k\frac{|(+e)\,q|}{(x-x')^2}\left(+\hat{i}\right)$$

Since the charge q is in equilibrium state so the total force exerted on it must be zero

$$\vec{F}_{tot} = 0 \;, \text{ equilibrium condition}$$

$$k\frac{|(-e)\,(+e)|}{x'^2}\left(-\hat{i}\right) + k\frac{|(+e)\,q|}{(x-x')^2}\left(+\hat{i}\right) = 0$$

$$\Rightarrow \frac{e}{x'^2} = \frac{|q|}{(x-x')^2} \Rightarrow e\left(x-x'\right)^2 = |q|\,x'^2$$

$$e(4\text{ nm} - 1\text{nm})^2 = |q|\,(1\text{nm})^2$$

$$\Rightarrow \;|q| = 9e$$

If we assume that the charge q is negative, we get the same result.

More Problems about all topics of electrostatic are also provided here.