1. **A missile is shot horizontally from the top of a 500 m cliff with an initial speed of** 300 m/s

   (a) Find the time it takes for the missile to hit the ground.

   (b) What is the range of the missile?

   (c) Calculate the velocity of the missile just before it hits the ground.

   (d) If the missile hits the ground and bounces up at an angle of 30° with a speed of 200 m/s, how far away from the point of impact will it land?

   Solution: This is a projectile motion problem, a type of motion in which, without air resistance, we have $a_x = 0$ and $a_y = -g$.

   The kinematic equations for projectile are:

$$
\begin{aligned}
x &= \underbrace{(v_0 \cos\alpha)}_{v_{0x}} t \\
y &= -\frac{1}{2}gt^2 + \underbrace{(v_0 \sin\alpha)}_{v_{0y}} t + y_0 \\
v_x &= \underbrace{v_0 \cos\alpha}_{v_{0x}} \\
v_y &= \underbrace{v_0 \sin\alpha}_{v_{0y}} - gt
\end{aligned}
$$

   (a) If we choose the initial position of the missile as the origin of the coordinate system (i.e. $x_0 = y_0 = 0$)then the hitting position has coordinate $(x, y = -500\,\text{m})$. Use the following kinematic relation to find the total flight time.

$$
\begin{aligned}
y &= -\frac{1}{2}gt^2 + v_0 t \sin\alpha + y_0 \\
\to \ -500 &= -\frac{1}{2}(9.8)\,t^2 + 300\,t \sin 0^\circ + 0 \\
\Rightarrow t_{tot} &= \sqrt{\frac{1000}{9.8}} = 10.10\,\text{s}
\end{aligned}
$$

   Note: in horizontally shot $\alpha = 0^\circ$ (b) To find the range of the projectile, we must find the total flight time of the motion and then substitute it into $x = v_0 t \cos\alpha$. In the previous part the total flight time is calculated as $t = 10.10\,\text{s}$, therefore

$$
\begin{aligned}
R_1 = x &= v_0 t \cos\alpha \\
&= 300 \times 10.10 \times \cos 0^\circ \\
&= 3000.30 \quad \text{m}
\end{aligned}
$$

   (c) In projectile motion first find the components of the velocity then use the $v = \sqrt{v_x^2 + v_y^2}$ to determine the velocity of the missile at any moment (in this case the hitting position, t_{tot}).

$$
v_x = v_0 \cos\alpha = 300 \times \cos 0^\circ = 300\,\text{m/s}
$$

$$\begin{aligned} v_y &= v_0 \sin\alpha - gt \\ &= 300 \times \sin 0^\circ - (9.8 \times 10.10) \\ &= -98.98\,\text{m/s} \end{aligned}$$

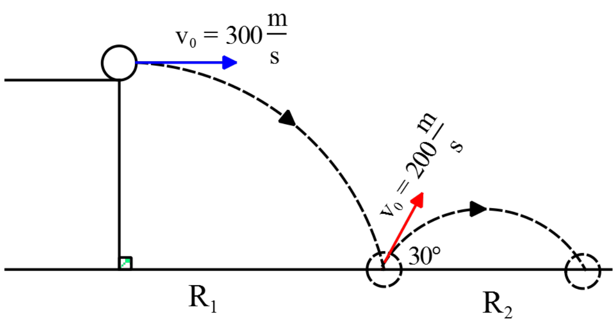
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(300)^2 + (-98.98)^2} = 315.9\,\text{m/s}$$

(d) In this part we have a separate projectile motion problem, hence we choose the launching point as the origin with the following information $x_0 = y_0 = 0$, $v_0 = 200$ m/s , $\alpha = 30^\circ$. First by setting $y = 0$ in the $y = -\frac{1}{2}gt^2 + v_0 t \sin 30^\circ \ + y_0$ find the total fight time, then substitute it into the $x = v_0 t \cos 30^\circ$

$$0 = -\frac{1}{2}(9.8)\,t^2 + 200\,t\left(\frac{1}{2}\right) + 0 \Rightarrow t\,(9.8t - 200) = 0$$

$$\Rightarrow \begin{cases} t_1 = 0\ , & \text{initial time} \\ t_2 = \frac{200}{9.8} = 20.4 \text{ s}\ , & \text{landing time} \end{cases}$$

$$\begin{aligned} R_2 = x &= v_0 t_2 \cos 30^\circ \\ &= 200 \times 20.4 \times \left(\frac{\sqrt{3}}{2}\right) \\ &= 3533.38\,\text{m} \end{aligned}$$



2. **A projectile is fired horizontally from a gun that is $45\,\text{m}$ above the ground. The muzzle velocity is $250\,\text{m/s}$.**
   - **(a) How long does the projectile remain in the air?**
   - **(b) At what horizontal distance from the firing point does it strike the ground?**
   - **(c) What is the magnitude of the vertical component of its velocity as it strikes the ground?**

Solution: (a) "Remains in the air" means the total flight time. To find this time put the coordinate of the impact of the projectile into the $y=-\frac{1}{2}gt^2+v_0t\sin\alpha\ +y_0$ and then solve for the time t_{tot}. Let the origin of the coordinates be the firing point. Therefore, the projectile hits the ground -45 mbelow the origin!

$$y=-\frac{1}{2}gt^2+v_0\sin\alpha\,t+y_0$$

$$-45=-\frac{1}{2}(9.8)t^2+250\,t\sin 0^\circ+0$$

$$\Rightarrow t_{tot}=\sqrt{\frac{90}{9.8}}=3.03 \quad \text{s}$$

(b) Namely, find the range of the projectile. Hence substitute t_{tot} into the x component of the motion i.e.

$$\begin{aligned}R=x&=v_0\cos\alpha t_{tot}\\&=(250)(3.03)\cos 0^\circ\\&=757.5 \quad \text{m}\end{aligned}$$

(c) The components of the velocity vector in a projectile motion at any moment in time are

$$\begin{aligned}v_x&=v_{0x}=v_0\cos\alpha\\&=250\cos 0^\circ=250 \quad \text{m/s}\end{aligned}$$

$$\begin{aligned}v_y&=v_{0y}-gt\\&=v_0\sin\alpha-gt\\&=250\sin 0^\circ-(9.8)(3.03)\\&=29.69 \quad \text{m/s}\end{aligned}$$

Where we have substituted the total time (the moment of impact on the ground).

3. **A ball is thrown horizontally from the roof of a building** $50-\text{m}$**-tall and lands** $45\,\text{m}$ **from the base. What was the ball's initial speed?**

Solution: This is a projectile motion problem with launch angle $\alpha=0^\circ$, so the projectile equations which are the x and y components of velocity and displacement vectors are written as below

$$x = v_{0x}t = v_0 \cos \alpha t$$

$$y = -\frac{1}{2}gt^2 + \underbrace{v_0 \sin \alpha}_{v_{0y}} t + y_0$$

$$v_x = v_0 \cos \alpha$$

$$v_y = \underbrace{v_0 \sin \alpha}_{v_{0y}} - gt$$

If we choose the releasing point as the reference then the coordinate of the point of impact is $(x = 45, y = -50 \text{ m})$. First, find the total flight time, then substitute it into the x component of the projectile.

$$y = -\frac{1}{2}gt_{tot}^2 + \underbrace{v_0 \sin \alpha}_{v_{0y}} t_{tot} + y_0$$

$$\Rightarrow \ -50 = -\frac{1}{2}(9.8)t_{tot}^2 + v_0 \sin 0° t_{tot} + 0$$

$$\Rightarrow t_{tot} = \sqrt{\frac{2 \times 50}{9.8}} = 3.19 \quad \text{s}$$

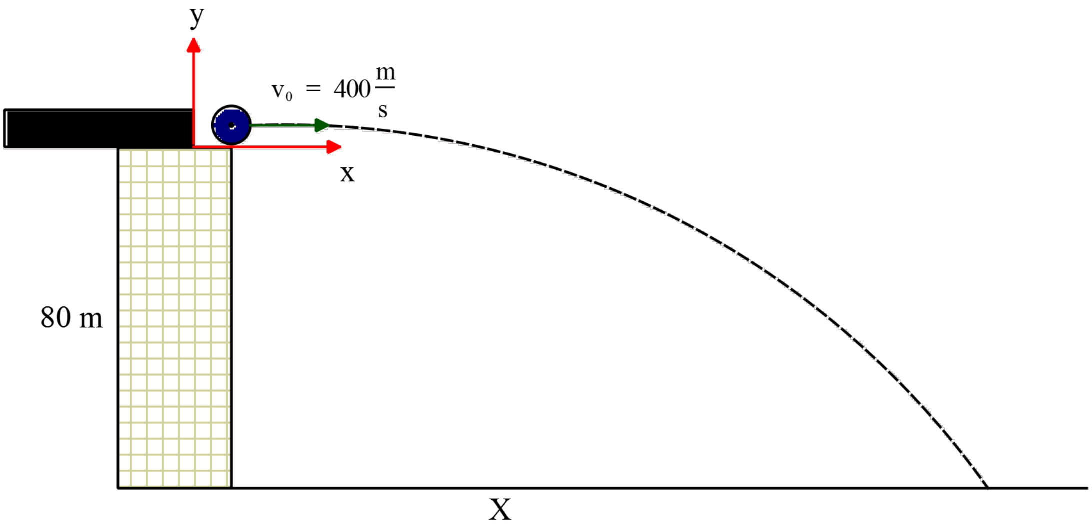
Therefore,

$$x = v_0 \cos \alpha t_{tot}$$

$$\Rightarrow v_0 = \frac{x}{\cos 0° \, t_{tot}} = \frac{45}{3.19} = 14.1 \quad \text{m/s}$$

4. **A** $1\,\text{kg}$ **projectile is fired from a cannon with an initial kinetic energy of** $10^4\,\text{J}$**. The cannon has an elevation angle of** $45°$**. How far does the projectile go before striking the ground (neglect the air resistance)?**

   Solution: In the projectile language, the distance from the launching to the striking point is called the range of the projectile that is found by substituting the total flight time into the x component of the projectile motion that is $X = v_0 \cos \theta \ t$.



From the definition of kinetic energy, one can find the initial velocity of the projectile below

$$K = \frac{1}{2}mv_0^2 \to 10^4 = \frac{1}{2}(1)\,v_0^2$$

$$\Rightarrow \boxed{v_0 = \sqrt{2} \times 10^2 \text{ m/s}}$$

Consider the starting and landing points to be on the same level. In this case, using the kinematic equation $v_y = v_0 \sin\theta - gt$ and knowing the fact that at the highest point, the vertical component of the projectile's velocity is zero, i.e., $v_y = 0$, find half of the total flight time that is $t_{tot} = 2t$ (since there is no air resistance).

$$v_y = v_0 \sin\theta - gt = 0$$
$$\to t = \frac{v_0 \sin\theta}{g} = \frac{\left(\sqrt{2} \times 10^2\right) \sin 45^\circ}{9.8} = 10.2 \text{ s}$$

This is the elapsed time to the highest point.

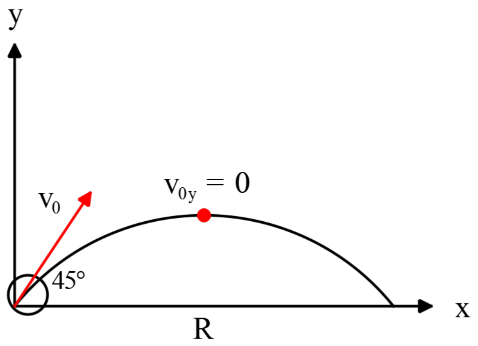
$$\Rightarrow t_{tot} = 2t = 20.4 \text{ s}$$

Therefore,

$$\begin{aligned} Range = X &= v_0 t \cos\theta \\ &= \left(\sqrt{2} \times 10^2\right) \cos 45^\circ \ \times 20.4 \\ &= 2040 \text{ m} \end{aligned}$$

5. **A bullet is fired horizontally from the top of a cliff which is** $80\,\text{m}$ **above a big lake. If the bullet muzzle (initial) speed is** $400\,\text{m/s}$**, how far from the bottom of the cliff does the bullet strike the surface of the lake? Neglect air resistance.**

Solution: A pictorial representation of the problem is shown in the figure below.



Put a coordinate at the starting point. Since the hitting point of the bullet is 80 m below the coordinate, so the coordinate of the landing point is ($x =?, y = -80\,\text{m}$). First, using the equation $y = -\frac{1}{2}gt^2 + v_{0y}t + y_0$ find the time required to reach the bullet to the ground.

$$y = -\frac{1}{2}gt^2$$

$$\rightarrow \quad t = \sqrt{\frac{2y}{-g}} = \sqrt{2 \times \frac{(-80)}{(-10)}} = 4\,\text{s}$$

Now using the relation between uniform velocity and displacement, i.e., $x = vt$ we obtain

$$x = vt = (400)(4) = 1600\,\text{m} \rightarrow \text{x} = 1\text{mile}$$